vides a way of attaining the maximal resources utilization while at the same time allows the direct specification of constraints in the primary objectives.

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A New Approach to Shock Isolation and Vibration Suppression Using a Resetable Actuator[1]

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A novel low power control technique along with a new class of actuators is developed for shock isolation and control of structural vibrations. In contrast to other techniques, including conventional viscous or rate damping, the force produced by the actuator has no velocity dependence. Several experimental, analytical, and simulation results are presented in support of this new, semi-active technique for structural control. The basic approach is to manipulate the system stiffness through the use of resetable actuators. With the proposed control approach, the actuator behaves like a linear spring. However, at appropriate times, the effective unstretched length of the actuator is changed—or reset—to extract energy from the vibrating structure. Experimental validation of the actuator model, analytical results on stability and actuator-placement, and simulation results for earthquake applications are presented. [S0022-0434(00)01603-8]

1 Preliminaries and Background

This work was originally motivated by the need for reliable actuators and control laws to suppress vibrations in structures caused by seismic excitations or wind loads. In earthquake applications, for example, the actuators could be unused for long periods of time, and then suddenly be called upon to produce extremely large forces. Consequently, active control laws may not be practical in such applications since large, expensive, hydraulic pumps must operate continuously to provide power to the seldom-used system. In such cases, techniques are needed that require low power and are highly reliable (through simplicity of design).

In many instances, the actuators and the corresponding control laws are designed such that no sizable amounts of energy can be added to the system and only a low power source is required to operate the control system. In this case, energy can only be taken out of the controlled system (hence the term ''semi-active''). A variety of techniques have been developed for many actuation mechanisms. Electrorheological, magnetorheological, and hydraulic devices (see [1–4] for a sample) have been developed for use on variable damping mechanisms, for which a variety of control laws (or logic) are developed. The primary mechanism used for control in most of these techniques is manipulation of the effective damping (i.e., rate of energy dissipation) in the overall system.

The concept of varying the stiffness of the structure has also been used in a variety of instances (see Utkin [5] as an early example). Unlike variable damping, in which increasing the effective damping can be accomplished rather easily by changing the orifice size, electric or magnetic fields, increasing effective stiffness may require substantial energy (e.g., when the spring elements are not at their unstretched position). Consequently, semi-active or low power implementation of variable stiffness techniques often requires additional constraints. Nemir et al. [6], Yang et al. [7], and Nagarajaiah [8] are representative of the papers dealing with the variable stiffness (active and/or semi-active implementation) technique. Note that manipulating stiffness can often yield large resisting forces with relatively simple hardware requirements.

In this paper, we discuss a new class of low power (i.e., semi-active) devices along with the corresponding control laws. The principle idea behind the approach is to manipulate the stiffness of the structure, by setting the effective stiffness of the element to high so that it can store energy. At appropriate times, the device is ''reset''; i.e., the stiffness is first reduced for a short time, which reduces the stored strain energy and then reset to the high value. After the reset has occurred, the stored strain energy is converted to heat. While a variety of mechanisms can be used for such an approach, the schematic shown in Fig. 1 is a model for the prototype actuators developed at UCI. This work was originally developed for hydraulic systems in Bobrow et al. [9]. In this paper, new results are developed on the design of gas actuators, on the placement of the actuators, and on disturbance rejection properties. More details can be found in Thai [10,11], and Srisamang [12].

In Fig 1, the piston and cylinder are connected to a one-degree-of-freedom system (Fig. 1(*a*)). The double acting cylinder is used to provide additional stiffness to the system. When the valve located between the two sides of the cylinder is closed, the fluid in the cylinder compresses as the mass moves. We will show that the net effect of the compression can be approximated by a linear spring, with an effective spring constant of k_1. By opening the valve for a short time and then closing it, it is possible to transform the potential energy stored in the fluid into heat. As a result, it also resets the unstretched length of the spring to whatever value x is when the valve is opened. Consequently, the mass experiences a stiffness of k_0+k_1 at all times, but the resetting of the actuator creates a forcing function in the equation of motion. This can also be seen by considering the potential energy in the system while writing the equations of motion. For the system in Fig. 1, the equation of motion is

$$m\ddot{x}+k_0x+k_1(x-x_s)=0. \qquad (1)$$

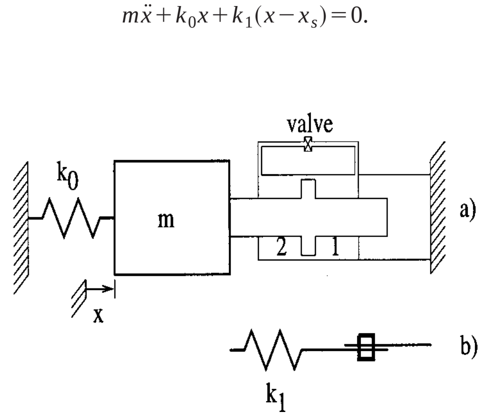

Fig. 1 Schematic of the active element

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In effect, the resetting of the device is equivalent to adjusting the unstretched length of the spring k_1. The basic hardware used here (and much of the analysis and control logic developed) can also be used for the variable stiffness form of the proposed semi-active technique, where the valve is closed only when the spring element passes through zero the stretch position (see [11] for details). For brevity, we focus on the resetting approach in this paper, which has certain advantages over the variable stiffness approach. These include the fact that the resettable stiffness element is storing energy at all times (to be released at appropriate instances) and the fact that the notions of natural frequency and mode-shapes are still applicable, even though the system is nonlinear (i.e., the homogeneity property discussed in Inaudi [13]).

The control law discussed below is to open the solenoid valve for a short time whenever the energy stored in the cylinder has reached a peak value. At these times, maximum energy is transferred from the vibrating system into heat in the actuator. As a result, the resetting logic—which can easily be extended to the multi-degree of freedom case—is the following: reset the actuator whenever $\dot{x}=0$. This control logic is based on the assumption that extraction of energy in very short time interval is practical.

There are several hardware designs that could be used to obtain the basic features of the actuator described in this paper. The choice of the actuator used depends on the stiffness requirements. A simple and reliable actuator can be obtained by using a gas such as air as the working fluid in the actuator. Air is easy to work with and, as shown below, can achieve a wide range of effective stiffness k_1 values. We first determine the relationship between the cylinder dimensions and the stiffness k_1. Let the pressure on the left-hand side of the actuator in Fig. 1 be p_2, and the pressure on the right-hand side be p_1. Assuming an ideal gas with no heat transfer through the cylinder walls, the pressures on both sides of the cylinder are governed by isentropic compression $pV^\gamma=c$ where γ is the ratio of specific heats, (γ=1.4 for air), V is the volume on one side of the cylinder, and c is a constant. Assuming we start motion from the mid-stroke position with the initial pressures on both sides of the cylinder equal to p_0 and initial volumes V_0, with $p_0V_0^\gamma=c$, we have

$$F(x)=(p_2-p_1)A=[(V_0+Ax)^{-\gamma}-(V_0-Ax)^{-\gamma}]Ac, \qquad (2)$$

where we have used $pV^\gamma=c$ on each side of the cylinder, and let the volume change with the cylinder position x. A local approximation for the effective spring constant of the gas and cylinder is obtained if one linearizes (2) for small motions of x. The result is

$$F(x)=-\frac{2A^2\gamma p_0}{V_0}x. \qquad (3)$$

Hence the effective spring constant is $k_1=2A^2\gamma p_0/V_0$. This relationship has been tested in Srisamang [12], where the linear approximation was compared experimentally to measured nonlinear data. The linear relation in (3) matched the nonlinear data over a range of about 60 percent of the cylinder stroke. In addition, tests were made regarding the cylinder reset time. It was found that the solenoid/cylinder combination tested took approximately 20 milliseconds to discharge to the point where $p_2\approx p_1$. This discharge time is fast enough to control systems with frequencies up to about 20 Hz. Systems with higher frequencies would require faster valves and larger orifices to achieve faster reset rates.

2 Analysis of One-Degree-of-Freedom Systems

In this section, we briefly discuss the motivation for the resetting approach. While most of the results presented here are later generalized to multi-degree-of-freedom systems, a single-degree-of-freedom system can be used to better describe the benefits of the proposed approach. As mentioned earlier, the control law for the system (1) is

$$\text{set } x_s=x \text{ whenever } \dot{x}(t)=0, \quad \text{and} \quad |x|>\epsilon, \qquad (4)$$

where ϵ is a small positive constant that keeps the controller from opening the valve near equilibrium configurations. At the instants when $\dot{x}=0$, the energy and displacement of the actuator are at a maximum, so the energy is discarded at these times.

One can express the transient response of the unforced system analytically as follows (also see [9,10,13]). Let the initial x_s be set to zero and time t_1 be the first time the actuator is reset, i.e., $x_s(t_1)=x(t_1)$ and $\dot{x}(t_1)=0$. The motion for this system can be expressed as

$$x(t)=a_0\cos(w_n(t-t_0)+\phi) \quad \text{for } t_0\leqslant t<t_1 \qquad (5)$$

where $w_n=\sqrt{(k_0+k_1)/m}$, and ϕ and a_0 are constants that depend on the initial conditions. At the time of the first reset, $t=t_1$ and from the control logic, $\dot{x}(t_1)=0$. Hence, $\phi=-\omega(t_1-t_0)$, and $x(t_1)=a_0$. From t_1 until the next reset time t_2, the system becomes

$$m\ddot{x}+(k_0+k_1)x=k_1x_s(t_1) \qquad (6)$$

where, $x_s(t_1)=x(t_1)=a_0$. Taking the appropriate initial conditions into account

$$x(t)=\frac{a_0}{(k_0+k_1)}[k_0\cos(w_n(t-t_1))+k_1] \qquad (7)$$

After a half cycle, i.e., $(t_2-t_1)=\pi/w_n$, we have $\dot{x}(t_2)=0$ and another actuator resetting. Thus

$$x(t_2)=-a_0\frac{(k_0-k_1)}{(k_0+k_1)}. \qquad (8)$$

This process can be continued forward in time. In general, after resetting the actuator $n-1$ times the motion and the initial conditions for the next resetting time t_n can be expressed as

$$x(t)=(-1)^{n-1}a_0\frac{(k_0-k_1)^{n-2}}{(k_0+k_1)^{(n-1)}}[k_0\cos(w_n(t-t_{n-1}))+k_1] \qquad (9)$$

for $t_{n-1}<t\leqslant t_n$, and so on. Since $q=(k_0-k_1)/(k_0+k_1)<1$, as $n\to\infty$, we have $x(t_n)\to 0$. Therefore the amplitude of the system decays exponentially with each half cycle. Note that the rate of decay depends on the fraction q. As k_1, the spring constant of the actuator increases from zero to k_0 the value of q decreases from 1 to zero, therefore the rate of decay increases. At the critical value of $k_1=k_0$, $q=0$, and the system reaches the desired equilibrium state one half cycle after the first piston reset with $t=t_1+\pi/w_n$. It is easy to show that if the variable stiffness approach was used (i.e., the valve is not closed immediately, but at the next instant of crossing the zero stretch position), the decay rate would be $\bar{q}=(k_1)/(k_0-k_1)$, which can be significantly slower than the resetting approach. Also, note that the natural frequency is not changed in the resetting approach, while the variable stiffness approach would have resulted in two distinct alternating regimes (one corresponding to low stiffness and the other to the high stiffness).

3 One-Degree-of-Freedom Example: Shock Absorber

As an application of the previous analysis, consider an example where $m=1$, and $k_0=100$ in a consistent set of units. We show three responses in Fig. 2, all with the initial conditions $x=-1$ and $\dot{x}(0)=0$. The undamped response, is the dash-dotted line with $\omega=\sqrt{k_0/m}=10$ rad/second. The solid line shows the response obtained with the new control law. In this case, the constants were $k_0=k_1=100$, so that $q=0$ from the above analysis. In this case it is assumed that the system is initially reset with $x_s=-1$. Note that the system settles after one half cycle as predicted above. Also shown for reference is the response (dashed line) that would be achieved with a viscous damper with damping ratio ζ=0.707. Note that the ideal damper cannot achieve settling times faster than the resetting type of control law.

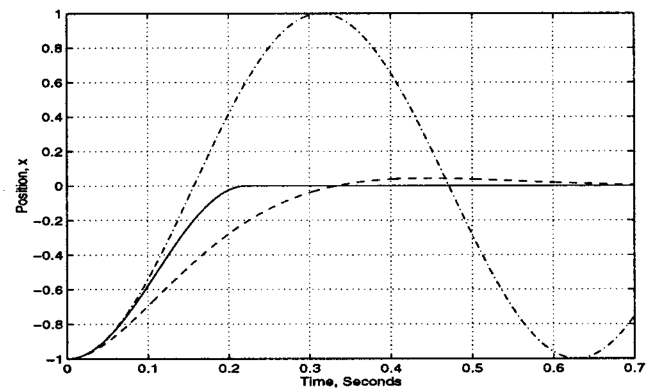

Fig. 2 Response of a mass-spring system. Dash-dotted plot is the undamped motion, dashed plot is viscous damping with ζ=.7, and the solid plot is the resetting control law.

4 Control Law for Multiple-Degree-of-Freedom Structures

This concept can be generalized easily to multi-degree-of-freedom systems with multiple devices as shown in Fig. 3 by considering the energy stored in the l actuators via

$$U_a = \frac{1}{2}\sum_{i=1}^{l} (x - x_{s,i})^T K_i (x - x_{s,i})$$

where K_i is the stiffness matrix associated with the ith actuator (in Fig. 1, K_i would be rank 1, but different mechanisms might have a more complicated structure for K_i). Here, x is the vector of generalized coordinates and $x_{s,i}$ is the piece-wise continuous vector denoting the zero force position of the ith actuator (i.e., the value of x at the last resetting of the actuator). Using this expression in the total potential energy of the system yields the following equations of motion

$$M\ddot{x} + Kx + \sum_{i=1}^{l} K_i(x - x_{s,i}) = 0, \tag{10}$$

where M and K are the nominal mass and stiffness matrices of the structure (i.e., without the resettable actuators). To develop the logic for switching (i.e., instances when the valve is opened to release energy and closed quickly to recover the original stiffness), we consider the total energy of the system; i.e., the structure plus the actuator/braces:

$$E = \frac{1}{2}[\dot{x}^T M \dot{x} + x^T K x] + \frac{1}{2}\sum_{i=1}^{l} (x - x_{s,i})^T K_i (x - x_{s,i})$$

$$= V + \sum_{i=1}^{l} U_i(t). \tag{11}$$

The first two terms are the kinetic and potential energy in the structure (which will be used as a Lyapunov candidate) and the last term is the energy stored in the resettable actuators.

Since the left-hand side of (11)—in the absence of damping—is constant when the actuators are locked (i.e., valves are closed), energy is moved from the structure to the actuators and back. As a result, the resetting logic is chosen to reset (i.e., release all stored energy in) each actuator when the maximum amount of energy is stored in it. This prevents the transfer of the stored energy back to the structure. As a result, the control logic can be summarized as

$$\text{set } x_{s,i} = x \text{ whenever } \dot{U}_i = 0, \quad \text{for } i = 1,2, \ldots l \tag{12}$$

where U_i is the energy in the ith actuator; i.e,

$$U_i = \tfrac{1}{2}(x - x_{s,i})^T K_i (x - x_{s,i}) \rightarrow \dot{U}_i = \dot{x}^T K_i (x - x_{s,i}).$$

The control logic in (12) appears to need all of the states (x and $\dot{x}$ vectors). In practice, however, the stiffness matrix associated with the ith actuator, K_i, is low rank and (12) can be simplified. As mentioned earlier, the stiffness associated with the actuator shown in Fig. 1 will be rank 1. Let z_i denote the relative displacement of the ends of the ith such actuator, and $z_{s,i}$ its value at the last resetting. It is relatively easy to show [11] that for the actuators discussed here, the control law becomes

$$\text{set } z_{s,i} = z_i \text{ whenever } \dot{z}_i \alpha_i (z_i - z_{s,i}) = 0 \tag{13}$$

where α_i is the stiffness of the ith actuator. The last equation shows the similarity between the control law for MDOF systems and the SDOF discussed earlier. Also note that the control law requires local measurement only and is thus decentralized. Since the mass and stiffness properties of the structure do not play an explicit role in the control law, the control law is robust with respect to modeling errors in mass and stiffness properties.

4.1 Stability and Actuator Placement. For the control law in (12), it is straightforward to show that

$$\lim_{t\to\infty} S_i(t) = 0 \tag{14}$$

where

$$S_i(t) = \dot{x}^T(t) K_i \dot{x}(t).$$

This is accomplished in several steps. In the first step, the total mechanical energy of the structure (but not the energy stored in the additional stiffness elements) is used as a candidate Lyapunoy function (V). Taking the derivative of this Lyaponuv function along the solution of (10), yields $\dot{V} = -\Sigma\,\dot{U}_i$. The resetting logic ensures that each actuator only absorbs energy so $U_i(t) \geqslant 0$ and is increasing, therefore, $\dot{U}_i \geqslant 0$ (note that U_i is reset to zero at the instant $\dot{U}_i = 0$). Thus (12) yields $\dot{V} \leqslant 0$ which establishes Lyapunov stability and boundedness of the state vector. Next, following standard arguments in invoking the LaSalle's theorem, it can be shown that $\lim_{t\to\infty}\dot{x}^T(t)K_i(x(t) - x_{s,i}) = 0$. Transforming to the local coordinates z_i, yields $\dot{z}_i = 0$ which implies (14). The details are straightforward and are omitted for brevity.

Consequently, the state of the system will be steered to the intersection of $S_i(t) = 0$. In presence of structural damping, the

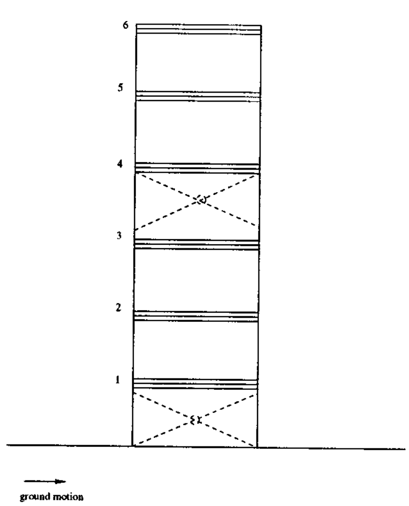

Fig. 3 Schematic of a six story building

state is steered to the intersection of the $S_i(t)$ and $S_c(t) = \dot{x}^T(t) C \dot{x}(t)$, where C is the damping matrix, i.e.,

$$\lim_{t\to\infty} x(t) \in S(t)$$

where

$$S(t) = \{S_1(t)=0\} \cap \{S_2(t)=0\} \ldots \cap \{\dot{x}^T C \dot{x} = 0\}$$

Furthermore, due to the homogeneity (see Inaudi et al. [13]), concepts of mode shapes and natural frequencies can still be used, even though the system is nonlinear. This is due to the fact that the stiffness matrix for the system does not change and the resetting of the actuators only affects the right-hand side of Eq. (10) through the $K_i x_{s,i}$ terms. This fact can be useful for developing ''hybrid'' control approaches in which a combination of semi-active devices and active control techniques are employed.

The model in (10) can be diagonalized through a state transformation, using the mode shapes associated with the M and K_0 matrices. These can be used to study the effect of the location of the elements and to develop general guidelines for actuator placement. For example, let q_i be the ith mode shape of the nominal structure; i.e., with stiffness K_0. Use of the standard transformation $x(t) = Qy(t)$, where Q contains the mode shapes and $y(t)$ is the modal coordinates, yields the following for each modal coordinate

$$m_j \ddot{y}_j(t) + \lambda_j y_j(t) = q_j^T \sum_i K_i q_j (y_j(t) - y_{s,i_j})$$

where m_j and λ_j are generalized mass and stiffness for the jth mode. The stiffness elements can be modeled as $K_i = v_i v_i^T \alpha_i$-where α_i is the stiffness of the actuator and v_i is the vector that depends on the placement of the actuator. Then it is easy to see that $q_j^T v_i \alpha_i$ plays a role similar to the the k_1 in (1) for the single degree of freedom discussed earlier. Also, by examining the modal decomposition, it is easy to show that $\lim_{t\to\infty} S_i(t) = 0$ implies that

$$\lim_{t\to\infty} v_i^T q_j y_j(t) = 0, j = 1,2, \ldots n.$$

Thus if $v_i^T q_j \neq 0$, then $y_j(t) \to 0$, but if $v_i^T q_j = 0$ then the ith actuator does not have any impact on the jth mode. Indeed, the larger the $v_i^T q_j$, the faster the vibration in the jth mode is eliminated. This can be used to help identify desirable locations for the actuators (e.g., placed such that highly excited modes are the ones most affected).

Generally, the results above establish that only the relative velocity, $\dot{z}_i(t)$, will go to zero as $t \to \infty$ (and not the whole state vector). Depending of the number and location of these actuators, the resulting system may not be asymptotically stable. However, if the inner product of every mode with at least one of the v_j is not zero, the whole state vector will go to zero.

It is also possible that combinations of modes will cause an actuator to reset when its displacement is at relative maxima, rather than the global maximum, during the motion of the structure. In this case, energy is absorbed at a slower rate than would be achieved if the control logic waited until the global maxima was reached. This modification would be difficult to implement in practice since one cannot distinguish between the relative maxima. Simulations conducted in [11] show that although resetting sometimes occurs before the maximum displacement has been reached, the immediate stabilizing effect of the energy absorption yields desirable damped-like behavior of the structural oscillations.

5 Conclusions

We developed a new control law and actuation approach to vibration suppression based on the concept of *actuator resetting*. The actuators used can be constructed from existing components at low cost. We developed a semi-active pneumatic actuator for this application and tested its force-position characteristics and its resetting capability experimentally. We have shown with Lyapunov methods that the variable structure control technique used will efficiently suppress vibrations in multiple degree of freedom structures. We also developed guidelines for the most effective placement of actuators in structural applications.

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Coding of Shared Track Gray Encoder

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A conventional absolute encoder consists of multiple tracks and therefore has a large size. Based on the theory of shared control, a new kind of absolute encoder called the shared track Gray encoder is proposed. This encoder has only one track, and the code has the characteristics of Gray code. In this paper, first the working principle of the encoder is introduced and the existence condition of the codes for the shared track Gray encoder is derived. Then the search procedure for the codes using self-developed assembler programs is given. Finally, the pattern of the

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