2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Optimal control and Performance of variable stiffness devices for structural control

John Leavitt, Faryar Jabbari and James E. Boborw

Abstract— **This paper addresses control of structural vibrations using semi-active actuators that are capable of producing variable stiffness. Usually vibration suppression is achieved using damping devices rather than variable stiffness ones. However, variable stiffness devices have significant advantages for shock isolation purposes. In this work we use a passivity approach to establish the requirements for the control law for a structure equipped with semi-active variable stiffness devices. We also solve a minimum-time, optimal control problem that demonstrates that our, passive, resetting feedback control law approximates the optimal control. An example simulation is given that shows favorable damping qualities are achieved for a three story building subject to the El Centro earthquake.**

I. INTRODUCTION

Recently, there has been a great deal of interest in actuation mechanisms that require low to negligible levels of power for operation. While the applications can be varied, here we focus our discussion on structural control, where traditional control approaches often dictate actuator forces that do not meet typical cost or reliability requirements. This has lead to mechanisms that produce sizable forces through manipulating structural characteristics (e.g., damping and stiffness), based on relatively simple control logic. Often, with a slight abuse of notation, these are called semi-active devices due to their very low power consumption (often provided by compact batteries) and they typically cannot add energy to the system (hence 'semi-active').

By now, there is extensive literature showing the benefits of this approach. While a comprehensive survey is not feasible, applications to structures and aerospace can be found [11] and [15], respectively, and applications to bridges and shock absorbers are presented in [16] and [4]. Typical early devices were hydraulic, but recent progress in electro-rheological and magneto-rheological material has lead to a variety of new semi-active devices based on these materials, which essentially manipulate the damping characteristics (see [5], [6], [7] and their references for a representative sample). Approaches to manipulate stiffness go back to variable stiffness models used in [18] or [12], in the context of variable structure control and quadratic stability, respectively, though the concept of semi-active (or low energy) was not present in such early work. The concept of semi-active stiffness devices was discussed in [14] and [5] and later in the work of [2]-[4] and [17] (the latter reference uses piezo material to develop variable stiffness devices that have a great deal in common, conceptually, with devices discussed here).

This work was supported by NSF grant CMS-0218813

Department of Mechanical and Aerospace Engineering, University of California, Irvine, CA 92697
Corresponding author: F. Jabbari, FJABBARI@UCI.EDU

Here, we focus on a new class of semi-active devices, introduced in [2]. These stiffness devices are capable of producing large resisting forces. The basic design is feasible for both pneumatic and hydraulic implementation, thus offers a great deal of reliability due to its reliance on standard hydraulic or pneumatic concepts, particularly when compared to devices employing novel material. Naturally, it possesses the low power, semi-active and decentralized properties that many of these devices share (see [20] for some of the benefits and advantages of the resetting devices, as compared to other semi-active approaches).

In section 2, after some preliminary discussion on the hardware including a sample of experimental results, we discuss the basic motivation from an optimal control view point. Next, for feedback operation in response to general disturbances, we examine the properties of the device through a passivity framework which naturally leads to a semi-active variable stiffness switching logic as well as a semi-active resetting logic. Both of these are then generalized for a generic multi-degree of freedom structural model, preserving their main properties (including the decentralized nature of the overall approach). In section 3 we show a set of representative results regarding their feasibility in structural applications.

II. PRELIMINARIES

The main idea is a device that acts like a spring, whose stiffness can be changed in real time. As discussed throughout this paper, we are interested in two forms: (i) when the stiffness of the device can be switched between zero and the maximum value at appropriate times (i.e., variable stiffness form), or (ii) when the stiffness can be changed from the maximum value to zero and *immediately* increased back to the maximum value (i.e, resetable from).

The basic concept can be demonstrated in the schematic shown in Figure 1, where we show a simple mass-spring system connected to the proposed device, which is depicted as a double-acting cylinder with an external line that connects the chambers on sides 1 and 2 of the cylinder through an on/off, or a proportional valve. When this valve is closed, motion of the piston compresses the gas, and as shown in [9], the force produced by the gas can be closely approximated by a linear spring with stiffness $k_1 = \frac{2A^2\kappa p_o}{v_o}$, where

A is the piston area, p_o is initial pressure, v_o is the initial volume, and κ is the ratio of constant pressure specific heat of the gas to constant volume specific heat (c_p/c_v) of the gas (see Figure 3 for a representative experimental plot). It was assumed for this derivation that p_o and v_o are equal on both sides of the piston. The linear spring approximation is represented in the Figure 1 below the cylinder as spring of stiffness k_1 connected to ground through a collar. When the valve is open, no force is produced by motion of the piston because the gas flows easily between the two sides of the cylinder. This corresponds to the collar being unlocked and sliding freely.

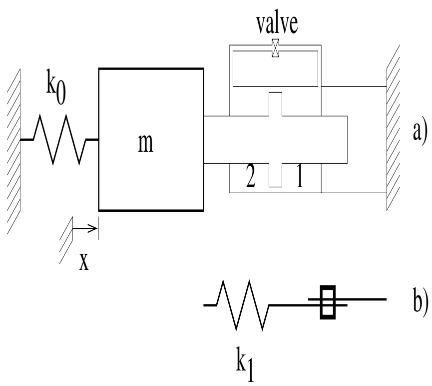

Fig. 1. Schematic representing the variable stiffness device.

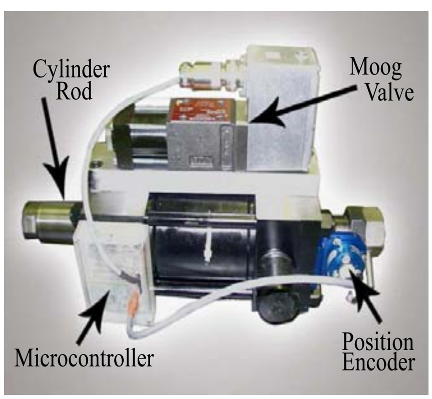

Fig. 2. Variable stiffness device capable of 30,000 lb output

Our hardware implementation of this device is shown in Figure 2. The cylinder is a standard Parker hydraulic cylinder capable of a peak pressure of 5000 psi with a 4 inch bore and a 3 inch stroke. The valve connecting the two sides of the cylinder is a Moog direct drive proportional valve capable of less than 5 ms response times with the orifice area proportional to the control voltage. We filled both sides of the hydraulic cylinder with nitrogen gas up to about p_o = 800 lb/inch^2 (55 atmospheres). Note that standard hydraulic cylinders can handle up to 5000 lb/inch^2, so that peak force level of about 30,000 pounds can be achieved with this actuator. Finally, the force-displacement

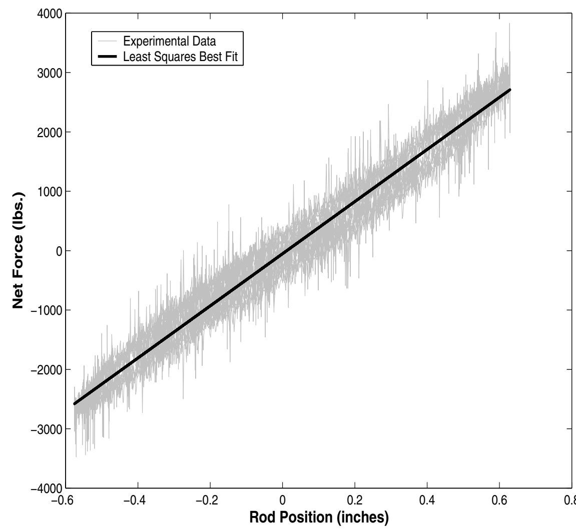

Fig. 3. Representative displacement vs reactive force for a variable stiffness device.

curve of our actuator is shown in Figure 3. In order to obtain this plot, the valve was closed and the actuator was oscillated while recording the pressures on the two sides of the cylinder along with the piston displacement. From the pressure data, the net force on the actuator could be determined. Although this pressure data is noisy, and there is a significant amount of friction present, the least-squares-fit plot shows that the nitrogen filled actuator behaves like a linear spring over the operating region of interest.

A. Optimal control of a gas-filled actuator

Given the ability to create any valve orifice area as the control $u_v(t)$ for this system, we first consider the solving following problem: "Which control extracts energy from the structure most quickly?" Given an initial condition, and assuming that a disturbance is known in advance, we can obtain a solution to this problem using tools from optimal control theory. Although it is generally not possible to know what the disturbance is going to be ahead of time, knowing the optimal solution to this problem sheds light on the form of the suboptimal feedback control law actually used.

In order to solve the optimal control problem, we first obtain the equations for motion for the structure and gas-filled actuator. For a single degree of freedom system like the one shown in Figure 1, the equations of motion are

$$m\ddot{x} = -k_o x + (p_2 - p_1)A, \tag{1}$$

where k_o is the structural stiffness, A is the area of the piston in the actuator, and p_1 and p_2 are the fluid pressures in chambers 1 and 2.

The dynamics of the gas flow and the chamber pressure are found by considering a power balance of the system [13].

$$c_p T\dot{m} - p\dot{v} + \dot{Q} = \frac{c_v}{R}\frac{d}{dt}(pv) \tag{2}$$

where p is the pressure inside the chamber, v is the chamber volume, $\dot{m}$ is the gas mass flow rate into the chamber, T is the gas temperature, R is the universal gas constant, $\dot{Q}$ is the heat transfer rate through the cylinder wall, and c_p, c_v are the gas constant pressure and constant volume specific heats, respectively. In (2), $c_p T\dot{m}$ is the internal energy of the air flowing into the chamber, $p\dot{v}$ is the power output by the moving piston, and $\frac{c_v}{R}\frac{d}{dt}(pv)$ is the time derivative of the total internal energy of the air in the chamber. We assume $\dot{Q} = 0$ because the heat transfer process has a much slower time constant than the air flow dynamics. We rewrite (2) by using $\frac{c_p}{c_v} \equiv \kappa$ and the fact $R = c_p - c_v$, to obtain to a differential equation for gas flow into chambers 1 and 2

$$\dot{p}_1 = \frac{\kappa}{v_1}(RT\dot{m}_1 - p_1\dot{v}_1) \tag{3}$$

$$\dot{p}_2 = \frac{\kappa}{v_2}(RT\dot{m}_2 - p_2\dot{v}_2). \tag{4}$$

The mass flow rates $\dot{m}_1, \dot{m}_2$ are controlled by the proportional valve. As shown experimentally in [13], the flow rates can be approximated reasonably well by

$$\dot{m}_1 = -\dot{m}_2 = cu_v(p_2 - p_1) \tag{5}$$

where c is a constant that depends on the valve orifice area, and u_v is the valve control voltage.

Equations (1)-(5) define the dynamics of the system, and given an initial condition and an excitation, the control $u_v(t)$ that extracts the energy from the structure in minimum time can be found. To accomplish this, we solved the following nonlinear optimal control problem:

$$\underset{u_v(t)}{\text{Min.}} \quad J(u_v(t)) = \frac{1}{2}\{kx(t_f)^2 + m\dot{x}(t_f)^2 + \int_0^{t_f} \epsilon u_v(t)^2\, dt\}, \tag{6}$$

subject to (1)-(5) and $u_v(t) \in [0, 1]$. With t_f fixed and ϵ a small positive constant, we are minimizing the energy in the structure at the final time. The nonzero weighting on the u_v^2 term in (6) was needed for the numerical algorithm used to solve the problem, but for ϵ small, it will not affect the solution significantly. For large t_f, the energy terms outside the integral are easily driven to zero. As t_f is decreased, at some point the energy terms can no longer be driven to zero for any control $u_v(t) \in [0, 1]$. The least time for which the energy terms can be driven to zero is the minimum time t_f^*, and the corresponding control $u_v^*(t)$ is the time-optimal control.

Figure (4) shows one example solution to the time-optimal control problem, note that the optimal $u_v^*(t)$ is usually zero, which means that the valve is usually closed so that no gas flows between the two chambers. But at instants when $x(t)$ is maximum or minimum, the optimal $u_v^*(t)$ pulses to one for a short time. The fact that the control is bang-bang in this manner is also a necessary condition for the optimal control. This is a standard result for minimum time problems for systems that are affine the the control (see e.g. [1]). Physically, this solution corresponds to keeping the valve closed until the gas in the actuator is most compressed, and opening the valve for a brief time so that the pressure equalizes between the two sides of the cylinder. In doing so, the maximum amount of energy is transformed from the vibrating structure into heat in the cylinder.

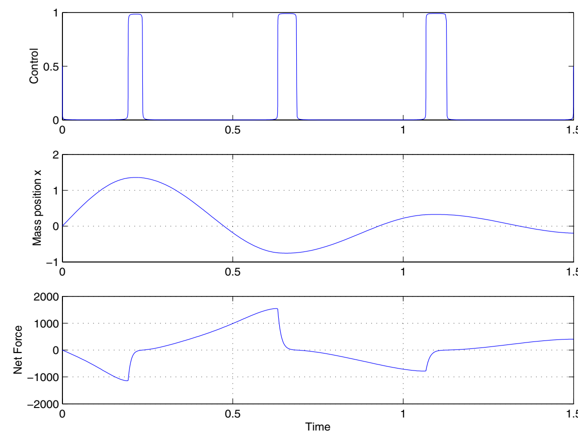

Fig. 4. u_v pulses for a short time while the actuator resets.

Inspection of the time-optimal control suggests that, for this application, there is a simple approximation using feedback; i.e., opening the valve at peak displacement and closing it soon after. As shown below, this is the ‘resetting’ technique mentioned in the introduction. In the next section, we will develop and analyze variable stiffness and resetting logics through the passivity framework, which will be applicable to general disturbances and m-d-o-f systems. For this, we use the linear approximation of a spring discussed in Figure 1 and Figure 3. First, we we examine the well-known *variable stiffness* technique.

B. Variable stiffness feedback control

In the variable stiffness technique, the device operates at two distinct stiffness levels. The valve can be opened (for low stiffness) at any time to extract energy, and it is changed to high stiffness (by closing the valve). In general, this leads to a system for which a variety of results from variable structure or switched systems can used. Note that, at a given deformation, increasing the stiffness of a spring requires the input of energy unless it is done at its unstretched position. Since we are interested in developing a low-power or semi-active device, this last issue plays an important role in developing control logic for this device.

We now discuss the model and corresponding switching law for the variable stiffness approach. In this case, the equations of motion are

$$m\ddot{x} + (k_o + \alpha(x)\, k_1\,)x + c_o\dot{x} = u(t) \tag{7}$$

where c_o is due to viscous friction (if applicable), $u(t)$ is an input or disturbance, k_1 the stiffness of the device, and $\alpha(x)$ is the switching law to be determined, which controls the stiffness provided by the device. While it is possible to develop devices that allow all stiffness values between zero and k_1 (i.e., $1 \geq \alpha(x) \geq 0$), we will focus on the case that we switch between the two extremes (i.e. $\alpha(x)$ is either zero or one). We rely on the passivity framework (see [10] for related definitions) and start with a storage function

$$V = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k_o x^2$$

i.e., the mechanical energy in the nominal system. It is easy to see that with velocity measurement, i.e., $y = \dot{x}$, we have

$$\dot{V} = -c_o y^2 + yu - \alpha(x) \; k_1 \dot{x} x.$$

The first two terms on the right hand side are from the nominal system and establish that the nominal system (i.e., without the devices) was strictly output passive (lossless if $c_o = 0$); i.e., the rate of energy storage in the system is less than (or for $c_o = 0$ equal to) the power injected by the input u. The last term is from the variable stiffness device. It is clear that in order not to alter the passivity of the system (e.g., to avoid increasing the stored energy at any time), thus satisfying the basic property of the semi-active approach, we need $\alpha(x) k_1 \dot{x} x \geq 0$. Given the range of values for $\alpha(x)$, it is clear that the desired semi-active switching law becomes

$$\alpha(x, \dot{x}) = 1 \quad \text{if} \quad \dot{x} x \geq 0,$$
$$\alpha(x, \dot{x}) = 0 \quad \text{if} \quad \dot{x} x < 0.$$

The desire to remove as much energy as possible yields an 'on-off' or two-state logic even if intermediate values of α were feasible. Note that the passivity properties of the nominal system is preserved and following standard steps, the L_2 or energy gain from u to y is bounded by $\frac{1}{c_o}$. Also, given that the storage function is positive definite, without external disturbances the system is asymptotically stable (for $c_o = 0$, a simple application of LaSalle's invariance principle will be needed).

Next, we generalize this approach to a multi-degree of freedom system, in which a number of these devices are installed. For small motion, x_i, the displacement *along* the length of the i^{th} device, can be represented by

$$x_i = T_i^T z$$

for some transformation T_i, where z is the vector of generalized coordinates and x_i is the motion along the main axis of the device. The energy stored in the device is thus

$$U_i = x^T T_i \; (k_i \alpha_i(z)) \; T_i^T x = \alpha_i(z) \; x^T K_i x$$

$i.e.$, $K_i = k_i T_i T_i^T$, with k_i the stiffness of the element and $\alpha_i(x)$ is the switching law. Ideally, we seek a decentralized switching law, i.e., $\alpha_i(x_i)$, which is possible as shown below. The equations of motion for the m degree of freedom structure become

$$M\ddot{z} + (K_o + \sum K_i \alpha_i(z)) z + C_o \dot{z} = Bu(t),$$

where B is the influence vector associated with inputs, while M and K_o are the nominal mass and stiffness matrices. Next, we define outputs $y = B^T \dot{z}$, and apply the same approach as before by using the positive definite storage function to be the mechanical energy

$$V = \frac{1}{2} \dot{z}^T M \dot{z} + \frac{1}{2} z^T k_o z.$$

which yields

$$\dot{V} = -\dot{z}^T C_o \dot{z} + y^T u - \sum \alpha_i(z) \; \dot{z}^T K_i z.$$

Recalling that $K_i = k_i T_i T_i^T$ and $x_i = T_i z$, we get

$$\dot{V} = -\dot{z}^T C_o \dot{z} + y^T u - \sum \alpha_i(z) k_i \dot{x}_i x_i.$$

As before, we obtain the same switching logic; i..e, to incorporate the semi-active nature of the problem, we need

$$\begin{array}{rcl} \alpha_i(x, \dot{x}_i) & = & 1 \;\; \text{if} \;\; \dot{x}_i x_i \geq 0, \\ \alpha_i(x_i, \dot{x}_i) & = & 0 \;\; \text{if} \;\; \dot{x}_i x_i < 0 \end{array} \tag{8}$$

which is decentralized and depends on local coordinates (i.e., motion along the length of the device) only, independent of nominal mass and stiffness properties.

If all modes are damped; i.e.,$C_o > 0$, we can write

$$\dot{V} \leq -\delta \, y^T y + y^T u - \sum \alpha_i(x_i) k_i \dot{x}_i x_i$$

where $\delta = \frac{\lambda_{min}(C_o)}{\lambda_{max}(B^T B)}$. Then standard passivity results show that the decentralized switching logic above preserves the estimate for the L_2 or energy gain from u to y (i.e., $\frac{1}{\delta}$), and asymptotic stability of the system (in the absence of external disturbances), under controllability or observability conditions that often met in practice.

As discussed in [9], when damping matrix is not positive definite, asymptotic stability is not necessarily guaranteed and the state vector converges to the intersection of sets or manifolds $\dot{z}^T C_o \dot{z} = 0$ and $\dot{z}^T K_i \dot{z} = 0$. In such cases, zero state observability with K_i or similar concepts may be used to establish asymptotic stability, though depending on C_o and location of the devices (i.e. structure of K_i) the system may be stable only.

Remark: The switching law above was developed by defining $y = B^T \dot{z}$, to exploit the passivity framework and establish the semi-active nature of (8). In practice, other (additional or different) outputs may be used for different purposes. For example, implementing the switching law in (8) requires, at a minimum, $\dot{x}_i$.

The switching law (8) has been used before. For example [18] used a similar logic for a single degree of freedom to demonstrate a simple variable structure system, while [11], [14] and [15] had used a variable stiffness devices to move energy to different modes, depending the excitation. In particular, [14] included a discussion on changing the stiffness to high values at zero deflections. More recently, the variable stiffness approach has been used by by Patten and co-workers (e.g., [16]) and Dawson and co-workers (e.g., [17], when the stiffness is altered with piezo-actuators). Typically, the stiffness is increased to the higher value according to a logic similar to (8)

Note however, that changing the stiffness to a higher value, when $x \neq 0$ requires the addition of energy, *if the model used is* (7). This is due to the simple observation that with both $\alpha = 0$ or $\alpha = 1$, (7) is a conservative system and a change in stiffness from a low value to a high value requires an injection of energy equal to $\frac{1}{2}(k_{high} - k_{low})x^2$. In practice, however, a variety of devices can be used to change the stiffness of the system at *any* position x, with little to no power. One such device is the gas filled actuator discussed in Section II-A. We now analyze the resetting approach using a passivity perspective.

C. Resetting control

In order to analyze the resetting approach, we start with the single degree of freedom case. At any given time t, we use x_s as the position of the piston at the last resetting of the device to its 'high' stiffness value; i.e., x_s is a piece-wise constant function, whose values are changed due to resetting. As a result, the energy stored in the device is $\frac{1}{2}k_1(x - x_s)^2$ since at any given time, the compression or extension of the spring is determined from the last resetting time. This leads to the equation of motion

$$m\ddot{x} + (k_o + \alpha(x)k_1)x + c_o\dot{x} = u(t) + \alpha(x)k_1x_s$$

where, as before, $\alpha(x)$ is either zero (low stiffness) or one (high stiffness). Using the same storage function as before

$$V = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}k_ox^2$$

and the output $y = \dot{x}$, we get the following

$$\dot{V} = -c_o\dot{x}^2 + yu + \alpha(x)k_1\dot{x}(x_s - x).$$

to preserve stability and performance (i.e., L_2 gain) of the nominal system, and to ensure a low power or semi-active structure to the device, we have the requirement

$$\alpha(x)\ \dot{x}\,(x_s - x) \leq 0\ ,\quad \forall\ \ t. \tag{9}$$

Naturally, if we guarantee that the stiffness is increased only at unstretched lengths, we ensure $x_s = 0$ in which case (9) becomes the variable stiffness case of (8). It relatively easy to see the following

- The control law in (9) is fail-safe. If at any time $\dot{x}\,(x_s - x) > 0$, by using $\alpha = 0$, we can ensure stability, passivity, etc. Note that $\alpha(x)\ \dot{x}\,(x_s - x) > 0$ implies that there is power flow into the storage function V, which is the mechanical energy of the nominal system (i.e., excluding the actuation devices). As such, the positive sign denotes energy transfer back to the structure from the devices (thus the need to set α to zero to remove the strain energy in the device before this can happen)
- At any time, α can be set to zero by opening of he valve. Since the energy stored is proportional to the square of the stretched length, it is desirable to delay resetting as long as possible. As a result, lowering the stiffness while (9) holds is not desirable.
- Consider a resetting event at $t = t^*$, when α is set to 1 and the value of $x_s - x$ is reset to zero (by definition). As a result, regardless of the sign of $\dot{x}$, we will have $\dot{x}\,(x_s - x) \leq 0$ for a period of time $t \in [t^*, \tilde{t}]$. To be precise, $\dot{x}\,(x_s - x) \leq 0$ will hold as long as $\dot{x}$ does not change its sign (from its value immediately after the most recent reset).
- Consequently, (9) requires reducing stiffness (setting $\alpha = 0$) if $\dot{x}$ is changing sign. The stiffness can then be increased (setting $\alpha = 1$) *at any time* afterwards.

Since the device is not collecting energy when $\alpha = 0$, it would be highly desirable to have have $\alpha = 1$ as much as possible. This leads to the following *ideal resetting* rule, which was the switching logic proposed in [9]:

$$\begin{array}{lll} \alpha = 0 & when & \dot{x} = 0 \\ \alpha = 1 & otherwise. & \end{array} \tag{10}$$

Using results from passivity, asymptotic stability can be shown (with or without damping) using LaSalle's theorem. When $c_o \neq 0$, the same estimate for the L_2 gain holds (i.e., $\frac{1}{c_o}$). Similarly, extension to multi-degree of freedom systems follows along the same steps as the variable stiffness case: The equations of motions (e.g., from Lagrange's equations) for a system with several devices will be:

$$M\ddot{z} + (K_o + \sum \alpha_i(z)K_i)z + C_o\dot{z} = Bu(t) + \sum \alpha_i(z)K_iz_{s,i}$$

Using $y = B^T\dot{z}$, and the same storage functions and transformation as before, we obtain the following decentralized control logic:

$$\alpha_i(x_i)\,\dot{x}_i\,(x_{s,i} - x_i) \leq 0\ ,\quad \forall\ \ t,\quad i = 1, 2, \ldots l. \tag{11}$$

Stability and L_2 of the system follow as before,and are thus not repeated.

Remark: Of the two techniques discussed here, the resetting approach is often more effective since it is collecting energy, to be drained at peak storage, at all times, whereas the variable stiffness device is 'off' roughly half the time. Also note that we have assumed continuity of the differential equations for the structural motion. This is a relatively mind assumption and is met in all realistic cases (to ensure chattering is avoided, one can introduce a small threshold in the control logic).

Remark In [19], the term 'Reset Control' is used to address a generalization of Clegg integral from the 1950's, which has shown benefits in improving overshoot properties of linear controllers. There are similarities between these approaches, in the sense the equations of motion here can be presented as a spacial case of the model used in [19], and the devices discussed here have shown strong overshoot suppression properties (see [3]). The Reset Control of [19], however, is a modification to a traditional (active) compensator, while the reset logic discussed here is vibration suppression device that is added to the structure or can be combined with a variety of other actuators, if desired (in which case the switched or hybrid systems approach might be an appropriate framework). Also, the passivity approach has led to stability and performance guarantees in relatively simple steps, consistent with the suggested future work in [19].

III. Performance Comparison

The main advantage of using a variable stiffness device for extracting energy from the structure, as opposed to damping devices, is that in cases of shock loading, large forces are not transmitted to the structure. This is because high velocities create large forces in traditional dampers, but create no force in the variable stiffness device (see [3] for an example application to an automotive suspension where the force transmitted through a conventional damper is more than an order of magnitude higher than the force transmitted through the resetting device). In this section we compare the performance of the resetting approach to that of an MR damper using the model developed in [5], where the NS component of the 1940 El Centro earthquake was the input to a three story structure. For the same structure, we simulate the results of placing a single resetable devices between the first and second floors. The device has an effective stiffness of about $9kN/cm$. In Table 1 we show the peak displacement (x_i) of each story relative to ground, the peak inter-story drifts (d_i), the peak absolute acceleration of each story (a_{ia}), and the peak force (f) for the uncontrolled systems as well as those obtained with either an MR damper or a resetable device.

As Table 1 shows, the performance of the two devices are quite similar, and both deliver significant improvements from the open-loop or uncontrolled case. This is not unexpected, since several studies (see [5] and references) have shown similar patterns; a relatively large number of devices with roughly equal capacity (e.g., maximum resistive force) showing more or less similar results. Generally, the resetable devices perform better at higher frequency disturbances (recall the discussions on their benefits in shock-type disturbances). Overall, these devices offer similar performance at far lower complexity (e.g., decentralized logic) with standard and reliable hydraulic technologies.

	Uncontr.	Clopped opt. (MR)	Resetting
x_1 (cm)	0.20	0.04	0.04
x_2 (cm)	0.31	0.07	0.08
x_3 (cm)	0.36	0.10	0.12
a_1 (cm/s^2)	421	341	363
a_1 (cm/s^2)	430	363	318
a_1 (cm/s^2)	571	341	340
f (N)	0	492	470
d_1 (cm)	0.20	0.04	0.04
d_2 (cm)	0.11	0.04	0.03
d_3 (cm)	0.05	0.03	0.03

TABLE I

Effects on a three story building

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