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Robotic Movement Training as an Optimization Problem: Designing a Controller that Assists Only as Needed

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Abstract—**One of the prevailing paradigms of physical rehabilitation following neurologic injury is to "assist-as-needed"; that is, the rehabilitation therapist manually assists patients in performing movements, providing only as much assistance as needed to complete the movement. Several research groups are attempting to automate this principle with robotic movement training devices. This paper derives an "assist as needed" robotic training algorithm by framing the problem as an optimization problem. We assume that motor recovery can be modeled as a process of learning a novel sensory motor transformation. The optimized robotic movement trainer is then an error-based controller with a forgetting factor. It bounds kinematic errors while systematically reducing its assistance. The same controller also works well if the dominant dynamics of recovery are akin to a strengthening process. We experimentally validate the controller with an unimpaired subject by demonstrating how the controller can help the subject to learn a novel sensory motor transformation (i.e. an internal model) with smaller kinematic errors than typical. The task studied here is walking on a treadmill in the presence of a novel dynamic environment. The assist-as-needed controller proposed here may be useful for limiting error during the learning of tasks in which large errors are dangerous or discouraging.**

I. Introduction

Robotic-assisted movement training following neurologic injury is a promising new field that seeks to automate hands-on therapy and promote neural recovery [1-4]. Currently, however, it is unclear how robots should assist in therapy in order to best promote neural recovery. Experienced rehabilitation therapists advocate "active assist exercise" or "assisting as needed", which refers to the principle of helping the patient perform a movement with the minimal amount of manual assistance possible [5]. Several robot control algorithms have been designed to automate active assist exercise, for both upper extremity and gait training [6-9]. However, these algorithms are currently ad hoc, unsupported by either rigorous modeling of the human motor system or by randomized, controlled, clinical tests. Developing these algorithms based on an understanding of the neural computations involved in adaptive control could provide a theoretical foundation for appropriate control strategies.

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In this paper, we formulate the "assist-as-needed" principle as an optimization problem. We assume that the robotic movement trainer must minimize a cost function that is the weighted sum of robot force and patient movement error. We then find the controller that minimizes this cost function for the case in which motor recovery is modeled as a process of learning a novel sensory motor transformation (i.e. learning an "internal model" of the impairment). We use an experimentally validated, computational model of internal model formation [10] that relates the perturbing force and previous kinematic error to predict the future value of that error. The resulting control law allows motor learning with small kinematic error, and systematically reduces its assistance as learning progresses. Thus, the controller "assists as needed". The same controller works well if the dominant dynamics of motor recovery are akin to a strengthening process. Finally, we present an initial experimental validation of this controller as a proof of the viability of concept.

II. Assistance-as-Needed as an Optimization Problem

To provide a context for the following controller derivation, assume that we are interested in designing a robotic control law for gait training. We would like the robotic device to assist in re-training the swing phase of gait. We quantify motor performance by step height x_i on the i^{th} step, and robot performance by peak upward force exerted R_i on the i^{th} step. We assume that the robotic movement trainer attempts to minimize a weighted sum of error and assistance force:

$$J = \frac{1}{2}(x_{i+1} - x_d)^2 + \frac{\lambda_R}{2}(R_{i+1})^2 \qquad (1)$$

where x_d is the desired step height and λ_R is a constant which weights the relative cost of the error and force terms. Notice that minimizing this cost function requires satisfying two competing goals: applying as little force as possible and making the person step as close to the normative step height, x_d, as possible. Thus, this cost function formalizes the principle of "assist-as-needed".

In order to find the controller that minimizes this cost function, we must model how the leg responds to applied forces. We assume that the patient adapts to a perturbing force field F_i applied to the leg on the i^{th} step with the following dynamics [10]:

$$x_{i+1} = a_0 x_i + b_1 F_i + b_0 F_{i+1} + c_o \tag{2}$$

These dynamics capture the process of internal model formation, which has been quantified in a wide range of experiments examining motor adaptation to imposed novel dynamic environments [10-12]. We have shown elsewhere that these dynamics minimize a cost function containing error, effort, and change in effort terms [10]. Further, they can be viewed as arising from the interaction of spring-like leg dynamics with the following muscle controller:

$$u_{i+1} = f_H u_i - g_H (x_i - x_d) \tag{3}$$

where u_i is force from muscular activity on the i^{th} trial, $f_H < 1$ is a forgetting factor, and g_H is the motor system's feedback gain for error-based correction of the muscle activity. Thus, our basic assumption about how the nervous system responds to an applied force is that it tries to model the force then counteract it, using an error-based learning controller. The parameters of (2) are related to the parameters of the controller as follows:

$$\begin{aligned} a_0 &= f_H - \frac{g_H}{K} \quad & b_0 &= \frac{1}{K} \\ b_1 &= \frac{-f_H}{K} \quad & c_0 &= (1 - (f_H - \frac{g_H}{K}))x_d \end{aligned} \tag{4}$$

where K is the limb stiffness. We assume now that the force field applied to the leg is the sum of two perturbations: the force applied by the assisting robot, R, and a virtual force created by a neurologic impairment, I_i:

$$F_i = R_i + I_i \tag{5}$$

The virtual force I_i can be viewed as the effect of the neural injury expressed as a force. For example, if the patient has difficulty lifting the leg following injury, we model this as the consequence of a virtual force that pushes the leg downward, relative to the normative condition.

Substituting (5) into (2) gives the dynamics of motor adaptation in response to the robot assistance and the impairment:

$$x_{i+1} = a_0 x_i + b_1 R_i + b_o R_{i+1} + b_1 I_i + b_o I_{i+1} + c_o \tag{6}$$

Now, the minimum of the cost function (1) occurs when:

$$\frac{\partial J}{\partial R_{i+1}} = (x_{i+1} - x_d)\frac{\partial x_{i+1}}{\partial R_{i+1}} + \lambda_R R_{i+1} = 0 \tag{7}$$

Rearranging (7) with the partial taken from (6) gives the robot controller that minimizes the cost function:

$$R_{i+1} = -\frac{b_0}{\lambda_R}(x_{i+1} - x_d) \tag{8}$$

Substituting (6) into (8) gives the error-based robot controller:

$$R_{i+1} = f_R R_i - g_R K(x_i - x_d) + c_R (f_H I_i - I_{i+1}) \tag{9}$$

with the following parameters:

$$\begin{aligned} f_R &= \frac{f_H}{\lambda_R K^2 + 1} \quad & c_R &= \frac{1}{\lambda_R K^2 + 1} \\ g_R &= \frac{f_H - \hat{g}_H}{\lambda_R K^2 + 1} \quad & \hat{g}_H &= \frac{g_H}{K} \end{aligned} \tag{10}$$

The robot controller (9) is thus similar to the human controller (3), in that it adjusts the robot force based on the step height error. It also uses a forgetting factor, f_R, to decrement the robot force on the next movement when error is small. The control law also contains a feedforward term related to impairment force I. This term is small if the impairment is assumed constant and the human forgetting factor is near one. One effect of this feedforward term is to initialize the robot force R so that it limits the initial kinematic error due to the impairment.

Here we assume that the impairment is a constant, so $\Delta I=0$. Combining the robot controller in (9) with the dynamics that describe motor adaptation (6) and substituting the RHS of (8) for R_i by shifting backward in time once gives the dynamics for the robotic movement training system as it interacts with the patient:

$$x_{i+1} = d_0 x_i + (1 - d_0)x_d + d_1 (I_{i+1} - f_H I_i) \tag{11}$$

with the following coefficients:

$$\begin{aligned} d_0 &= f_H - \hat{g}_H - g_R + (f_H - f_R)/\lambda_R K^2 \\ d_1 &= \frac{\lambda_R K}{\lambda_R K^2 + 1} \end{aligned} \tag{12}$$

Note that these dynamics arise from the interaction of two adaptive processes: the robot control algorithm (9), and the human motor adaptation (6) to the applied forces.

III. Analysis of the System Behavior

A. Simulation

We simulated the behavior of the system in order to verify the operation of the controller. We first examined how the system responded to an applied force field without robotic assistance. For simplicity and ease of presentation, we assume that the impairment force I = -1 N such that the impairment pulls the leg downward. Furthermore, we assume the following values for the learning gain, g_H = 0.3 N/m limb stiffness, K = 1 N/m, and forgetting factor, f_H = 1. Other values are certainly possible and can even be identified experimentally [13], but this set of values conveniently illustrates the essence of the system behavior.

For the simulations, we activated the impairment force at step number 25. Figure 1 shows the response of (6) with the robot force, R = 0. The impairment causes the step height to decrease at step 25. The subject then forms an internal model of the impairment, gradually increasing the muscle force to counteract it, and gradually reducing the step height error.

The modeled motor system compensates well for the impairment, so the question arises as to why robotic assistance would be needed. A critical assumption regarding the type of rehabilitation situation we are considering here is that impairments can induce large errors, which are often prohibitive to subject movement. Robotic therapies offer a solution to bound errors and thus restore movement ability. For example, in our current example of an impairment that affects step height, if step height were too low, then the patient would trip and be unable to step.

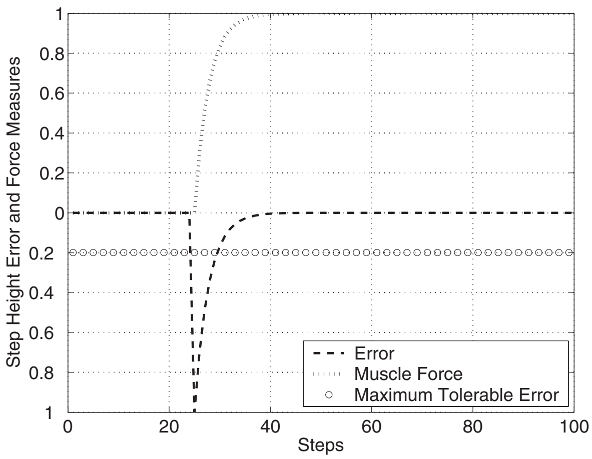

Figure 1. Behavior of Human Adaptive System to Imposed Impairment. Impairment occurs at step number 25 and causes a decrease in step height.

We denote an arbitrarily chosen maximum tolerable error of –0.2 with open circles (o) (FIG.1). Hypothetically, if the step height error were to go below this line, the subject's movement would be prohibited and thus he could not practice walking, and the motor system would not learn to compensate for the impairment.

Next, we add the robotic assistance (9) to the system. We choose λ_R = 0.01 in (1), thus placing emphasis on maintaining a small step height error. We assume that the robotic assistance and impairment both turn on at step 25. The effect of the robot controller is to bound the step height error, which would hypothetically allow the subject to practice moving, and thus to model the impairment force (FIG. 2). Notice that the time taken to learn the muscle force required to cancel the impairment force is longer than without assistance (cf. FIG. 1), because of the reduced movement error.

B. Optimality Constraints of the Control Gains

At the onset, one might have guessed that an error-based robotic assistance algorithm with a forgetting factor would "assist-as-needed". In fact, we previously tested this concept [14]. These previous simulations indicated that the ability of the controller to assist-as-need required that the robot forgetting factor f_R be less than the human forgetting factor, f_H; i.e. the robot must "out forget" the human system.

We gain analytical insight into this condition, as well as another key stability condition, by framing the problem as an optimization problem in the present paper. We express these insights as two observations:

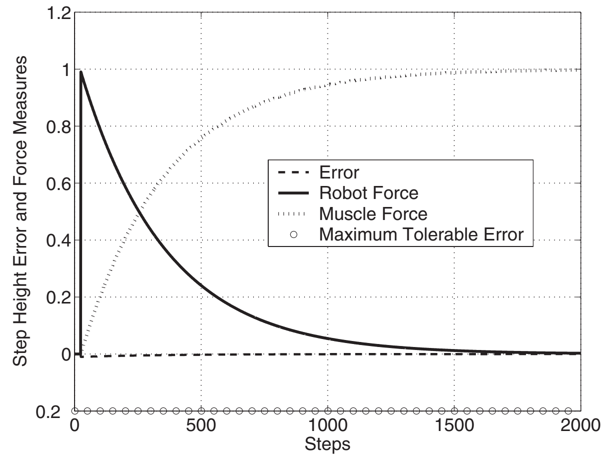

Figure 2. Behavior of Coupled Robot and Human Adaptive System. Impairment and Robot assistance occur at step 25. The robot bounds the error while still allowing learning.

Observation 1: The optimized assist-as-needed robotic assistance algorithm uses a forgetting factor, f_R that is less than the human forgetting factor f_H.

This observation stems from the examination of the equation for f_R in (10), which was derived assuming an optimized controller. Note that $f_R < f_H$ for all λ_R. Thus, the robot must attempt to decrease its force more quickly than the human controller in order to assist only as needed, as we found previously in simulation [14].

Observation 2: The optimized assist-as-needed algorithm is stable as long as the human adaptive controller is stable, because the algorithm bounds the error-based learning gain g_R.

The stability of the robotic system as it interacts with the human is determined by the coefficient d_0 in (12). In order for the interacting system to be stable $|d_0| < 1$. The robot controller selects g_R and f_R per (10). Substitution of these formulas into the equation for d_0 in (12), gives

$$d_0 = f_H - \frac{\lambda_R K^2}{\lambda_R K^2 + 1}\hat{g}_H . \qquad (13)$$

Thus $|d_0| < 1$ for all $\lambda_R > 0$ if $|f_H\text{-}\hat{g}_H| < 1$ and $0 < f_H < 1$. According to (2) and (4), the condition $|f_H\text{-}\hat{g}_H| < 1$ corresponds to the stability condition for the human adaptive system operating on its own. Thus, even though it is possible to select a large error-based learning gain g_R for the robot in (9), a large gain would make the coupled system unstable (even if the individual robot and human systems were stable). This situation is prevented because the optimized controller bounds the robot gain in (10) relative to the parameters that determine the stability of the human adaptive system.

IV. INCLUDING A MODEL OF WEAKNESS

We have assumed so far that the patient is capable of exerting any force required to counteract the effect of the impairment, and, consequently, that the primary issue in neural recovery is one of relearning an appropriate sensory motor transformation, i.e. an internal model of the effects of the injury. It is likely, however, that the neurologic injury will also affect the ability of the motor system to produce the forces required to implement an internal model. Weakness is a profound and common impairment following stroke and spinal cord injury. We can model the effect of weakness by including a constraint on the maximum muscle forces that can be exerted, by controlling the output of (3):

$$u_{i+1} = \min(u_{i+1}, S_{i+1}) \ , \tag{14}$$

where S, which we term "strength", is a measure of the maximum motor command that is capable of being implemented by the human motor controller. We further assume that the value of this maximum motor command will improve with practice toward full strength ($S = S_{max}$):

$$S_{i+1} = mS_i + (1-m)S_{\max} \ , \tag{15}$$

where m determines the time constant of overcoming the weakness. We have assumed, for presentation simplicity, that $S_{max} = 1$ and m = 0.98. Thus, we assume that the patient starts out weak and gains strength over time, in a first-order process driven by movement repetition. This assumption has the effect of increasing the amount of time taken to model the field (FIG. 3), as compared to (FIG. 1). Note again that error is below the maximum tolerable error of –0.2 and thus, hypothetically at least, the subject would be incapable of movement and therefore rehabilitation. This simulates the rational for why subjects enter therapy.

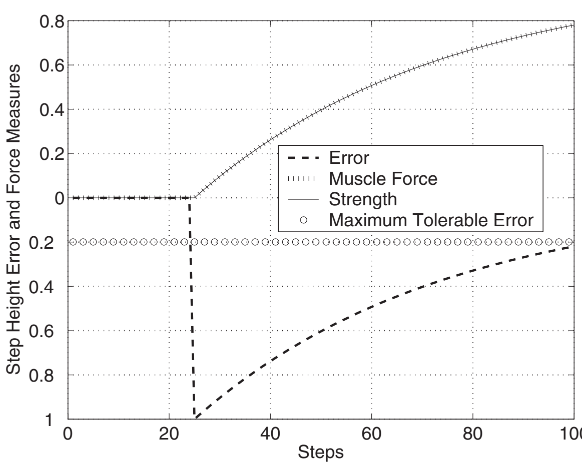

Figure 3. Behavior of Unassisted, Weakened Human Adaptive System. Strength is assumed to increase slowly according to (15). Impairment occurs at step 25. Here, muscle force and strength overlap indicating that recovery is limited by strength.

V. SIMULATION OF SYSTEM BEHAVIOR WHEN WEAKNESS DOMINATES THE RECOVERY DYNAMICS

We return now to the example of gait training to place the following simulation in context. Assume a person experiences a neurologic injury that not only alters their sensory motor transformation for controlling leg movement, but also greatly reduces leg strength. The reduced leg strength prevents walking, and thus the person never strengthens the leg or learns the altered sensory motor transformation. With robotic therapy, the robot can bound errors while assisting as needed, allowing the subject to both strengthen the leg and model the field. Figure 4 shows the situation where the impairment occurs at step 25 and large errors are present, which prevent stepping practice. At step 50, the patient enters robotic therapy with the robotic controller described by (9). The controller bounds the step height error. The patient gradually strengthens the leg, and, concurrently, gradually forms an internal model of the impairment. Thus the assist-as-needed controller also is successful in reducing error, promoting learning, and strengthening when strengthening dynamics are the rate-limiting process of rehabilitation (FIG.4). For comparison, Figure 4 also shows the effects of a controller that assists with a fixed level of assistance (i.e. $f_R = 1$). Note that while error is small, but the motor system never learns to model the force.

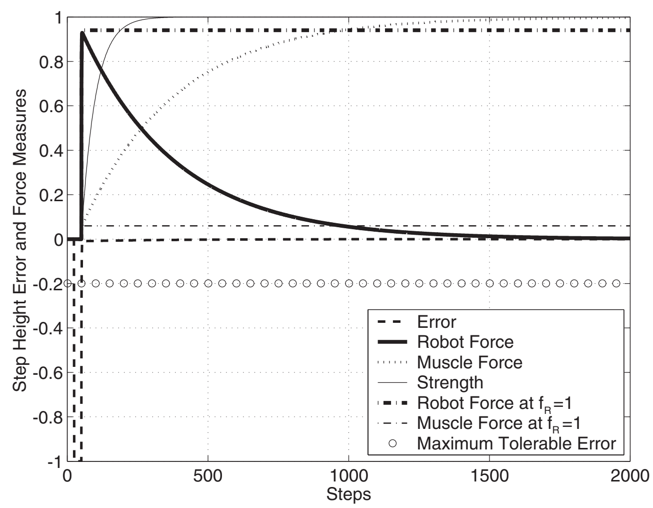

Figure 4. Time course of recovery to neurologic injury when strengthening dynamics dominate recovery and the robot assists as needed. Subject suffers impairment at step 25 and attempts to recover. Errors are large and prohibitive to movement. Subject enters robotic therapy with adaptive robot assistance (9) at iteration 50. The robot bounds errors, allowing subject to develop strength and model the impairment.

VI. EXPERIMENTAL VALIDATION FOR A LOCOMOTOR ADAPTATION TASK

We experimentally validated the assist-as-needed robotic assistance algorithm for the case of an unimpaired human subject adapting to a perturbing force field during walking on a treadmill. Here, we propose an analogy between the perturbing force field and a sensory motor impairment. We show that the subject can learn an internal model of the force field (i.e. the "impairment") with smaller kinematic errors when the robot "assists as needed."

A. Experimental Setup

A single healthy male subject completed two experiments during which a 2-DOF planar robot (described in [13]) applied an upwardly directed, viscous force field to the subject's lower shank during walking on a treadmill. The viscous force was proportional to the subject's forward velocity during the swing phase of gait (gain = 44 Ns/m). The effect of this force field was to push the leg higher than normal during swing. Our purpose was to compare how the subject adapted to the force field with and without robotic assistance. The position of the lower shank was collected at 200Hz. In a first 'baseline' experiment we collected data that allowed us to identify the parameters of the learning model (4), and from these parameters, the robot assistance parameters from (10). This baseline experiment consisted of 500 steps in five stages. In the first stage, the robot applied no force (null field) for 50 steps. This was followed by 200 steps in the null field during which the robot applied 30 randomly spaced "catch trials" for which the force was turned on for a single step with a random gain between 17 and 71 Ns/m. During the third stage, the robot applied a constant gain viscous force field, gain = 44 Ns/m for 50 steps. During the fourth stage of 100 steps with the previous gain, the robot applied 15 randomly spaced "catch trials" with a random gain between 17 and 71 Ns/m. The fifth stage concluded with 50 steps in the null field. During the second experiment, the subject adapted to the force field with and without robotic assistance. The subject stepped in a series of three stages. The first and third stage were standard tests of adaptation: 50 steps in the null field, followed by 50 steps in the force field, followed by 50 steps in the null field again. During the second stage, the subject stepped in the perturbing field for 200 steps, with a second robot force superimposed on the perturbing force field. Specifically, the robot provided a superimposed, assisting force field using the "assist-as-needed" robot control in (9). The robot control law determined the gain of this force field, which was of the same form (i.e. vertical viscous force) as the perturbing force. Following the initial 100 steps, ten randomly spaced "catch trials" were used to test for the presence of aftereffects and thus internal model formation.

B. Experimental Results

Subject specific parameters were obtained through multiple linear regression on data from the baseline experiment. The R^2 value for the subject was 0.68. Figure 5 shows the results of the second experiment with the weighting coefficient λ_R chosen to be 0.01. When the force field was applied in the first and third blocks (trials 50 and 400), the subject exhibited a large step height error, but then reduced the error with practice over the next ten to twenty steps. When the field was unexpectedly removed (step 100 and 450), the subject exhibited an aftereffect of adaptation, stepping lower than normal, indicating that he had formed an internal model of the force field. When robot assistance was provided according to the "assist-as-needed" control law (9) in the second block, the direct effect (step 150) was significantly smaller than without assistance ($p<0.03$), ttest comparing direct effect with robot assistance to two direct effects without robot assistance). However, the subject still exhibited comparable aftereffects when the force field was removed during catch trials (interspersed in steps 250 to 350).

Theoretical predictions for the evolution of robot assistance and step height error are confirmed by the experimental data as predicted by theory (FIG. 6). A key feature of the evolution of the robot assistance is that it slowly decreased as the subject learned the force field. Thus, the robot at first assisted the subject more in order to reduce step height error, but then gradually reduced its assistance as the subject learned to compensate for the force field on his own.

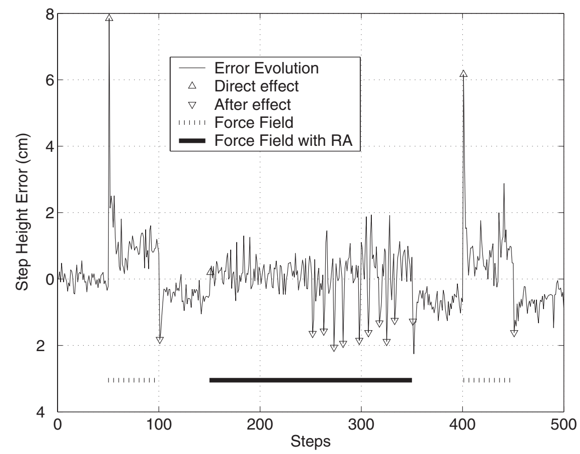

Figure 5. Step height error as a function of step number during the experimental validation of the assist-as-needed controller. The broken horizontal bars shown when the force field was applied without assistance-as-needed, and the solid line shows when the same force field was applied but also with a superimposed assist-as-needed force. RA is an abbreviation for Robot Assistance.

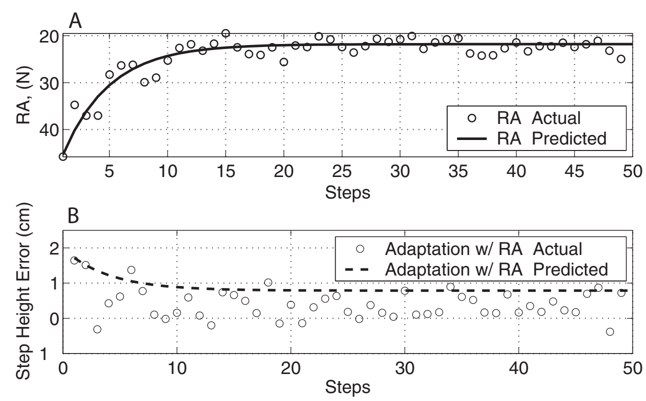

Figure 6. Experimental data overlaid with the theoretical predictions for the first fifty steps in the force field with robot assistance (i.e. steps 150-200 in Figure 5). A: Robot assistance (RA), quantified by the peak force per step of the assisting robot force field. B: Step height error during adaptation to the force field with (w/) Robot Assistance.

VII. DISCUSSION

By framing the assist-as-needed therapeutic goal as an optimization problem, we derived a robotic training algorithm that can bound errors while still allowing learning. The controller takes the form of an error-based learning controller with a forgetting factor, similar to the human motor controller itself. Such a controller works for the case in which the primary dynamics of recovery are that of learning a novel sensory motor transformation, or that of strengthening. The controller can be distinguished from previous robot therapy controllers that provide a fixed amount of assistance with impedance, force, or position controllers because it includes a "forgetting" process that reduces the applied force from movement to movement as a function of measured error.

We experimentally validated the assist-as-needed controller for the case of an unimpaired subject adapting to a perturbing force field during walking on a treadmill. The direct effect and subsequent kinematic errors were kept smaller with robot assistance, but the subject still learned an internal model of the force field as evidenced by the presence of comparable after effects following adaptation. To achieve learning with reduced error, the assist-as-needed controller gradually reduced the level of assistance it provided; this reduction was well predicted by the theory (Figure 6).

There are two key issues in the practical implementation of this technique for rehabilitation therapy. First, the controller requires knowledge of f_H, g_H, K, I, x_d and S. It is possible to identify f_H, g_H, K, and x_d for unimpaired subjects by measuring adaptation to a force field [13], but it is unlikely that such identification techniques would work for impaired subjects if their impairment limits their ability to perform the task. One possible solution is to use parameters identified for unimpaired subjects. More generally, the control law will still work even if the gains are not precisely chosen. Varying the gains has the effect of altering the weighting between force and error, i.e. λ_R. Thus, appropriate gains might be found by trial-and-error. The two observations presented here will guide the trial-and-error process: the robot must "out-forget" the human, and the robot error-based learning gain must not be too large.

The second issue is to translate the controller into one that controls a set of trajectory and force parameters, rather than just one parameter in one direction. We currently control for only one parameter of performance, in this case, step height. Control of the entire stepping trajectory might be accomplished with a feedback controller that intelligently alters the feedback gain in a forgetting fashion similar to (9). Alternatively it should be possible to expand this controller for a series of positions along a stepping trajectory, although K, g_H, f_H, S and λ_R might have to specified for each position along the trajectory.

VIII. Conclusion

These results provide a theoretical basis and an initial experimental validation for a class of robot movement training algorithms that reduce kinematic error, but still permit learning to occur, in a sort of "smart training wheels" effect. Controllers such as these may be important for allowing motor training after neurologic injury by bounding errors to movement trajectories while simultaneously reducing assistive forces when errors are small, thereby allowing learning to occur. A similar approach might be useful for precision motor skill training, such as during fine surgery or athletic training, to bound trajectory errors of novices while still allowing learning.

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