Optimal Performance of Variable Stiffness Devices for Structural Control

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This paper addresses control of structural vibrations using semi-active actuators that are capable of manipulating stiffness and/or producing variable stiffness. Usually vibration suppression is achieved using damping devices rather than stiffness ones. However, stiffness devices can produce large forces and have significant advantages for shock isolation purposes. In this work we use a passivity approach to establish the requirements for the control law for a structure equipped with semi-active stiffness devices. We also solve an optimal control problem that demonstrates that our passive, resetting feedback control law approximates the optimal control. Simulation and experimental results are presented in support of the proposed approach. [DOI: 10.1115/1.2432360]

1 Introduction

Recently, there has been a great deal of interest in actuation mechanisms that require low to negligible levels of power for operation. Although the applications can be varied, here we focus our discussion on structural control, where traditional control approaches often dictate actuator forces that do not meet typical cost or reliability requirements. This has lead to mechanisms that produce sizable forces through manipulating structural characteristics (e.g., damping and stiffness), based on relatively simple control logic. Often, with a slight abuse of notation, these are called semi-active devices due to their very low power consumption, often provided by compact batteries. Generically, the term semi-active often is used for devices that cannot add energy to the system.

By now, there is extensive literature showing the benefits of the semi-active control approach. Although a comprehensive survey is not feasible here, applications to structures and aerospace can be found in [1,2], respectively, and applications to bridges and shock absorbers are presented in [3,4]. Typical early devices were hydraulic, but recent progress in electrorheological and magnetorheological material has lead to a variety of new semi-active devices based on these materials, which essentially manipulate the damping characteristics (see [5–8], and the references within for a representative sample). Approaches that manipulate stiffness go back to variable stiffness models used in [9], in the context of variable structure control and quadratic stability, respectively, although the concept of semi-active (or low energy) was not present in such early work. The concept of semi-active stiffness devices was discussed in [8,10] and later in the work of [4,11–13] ([13] uses piezomaterial to develop variable stiffness devices that have a great deal in common, conceptually, with devices discussed here).

Roughly speaking, these devices act as additional stiffness elements that store energy during compression or elongation. By reducing the stiffness (e.g., opening a valve or releasing a locking mechanism), the stiffness is reduced suddenly, resulting a rapid loss of stored energy. The control logic is often aimed at finding suitable points for reducing the stiffness and then increasing it back to the high value. If the stiffness can be increased without energy input, the resulting device will meet the semi-active characterization.

Here, we focus on a new class of semi-active devices, introduced in [11]. These stiffness devices are capable of producing large resisting forces. The basic design is feasible for both pneumatic and hydraulic implementation, thus offering a great deal of reliability due to its dependence on standard hydraulic or pneumatic concepts, particularly when compared to devices employing novel materials. Naturally, it possesses the low power, semi-active, and decentralized properties that many of these devices share. More importantly, in addition to the traditional variable stiffness implementation, it can be used in a "resetting" arrangement that has many additional advantages (see [14] for some of the benefits and advantages of the resetting devices, as compared to other semi-active approaches).

Concepts similar to our resetting (originally developed in [11]) have been proposed by others. In [15], a version of this approach (though not necessarily the semi-active form) was studied, including the homogeneity property, in which the nonlinear system retains the same eigenvalues and eigenvectors. Similarly [2] considers an approach quite similar to our resetting approaches, but the logic is not decentralized and often depends on the large dimensional modal representations. Here, we discuss the resetting devices and techniques of [11]. Basic properties, particularly for structural applications, were reported in [16]. Recently, a benchmark problem was used for evaluation and comparison for different semi-active approaches (see [17] and references therein). More recently, large capacity devices have been developed (see [18]), whose effectiveness and reliability have been verified through full-scale testing [19]. While we review these results, briefly, our main focus in this paper is to present analytical results regarding the motivation (from optimal control) as well as stability and performance measured (based on passivity arguments). We thus avoid the often ad hoc (or strictly device based) approaches used in much of the semi-active field. For example, our results can easily accommodate the inevitable delays that are faced when the stiffness is to be increased.

In Sec. 2, we provide a preliminary discussion on the basic design. Section 3 deals with motivation from an optimal control viewpoint. Next, for feedback operation in response to general disturbances, we examine the properties of the device through a passivity framework which naturally leads to a semi-active variable stiffness switching logic as well as a semi-active resetting logic. Both of these are then generalized for a generic multidegree-of-freedom (MDOF) structural model, preserving their main properties, including the decentralized nature of the overall approach. In Sec. 4, we show a set of representative ex-

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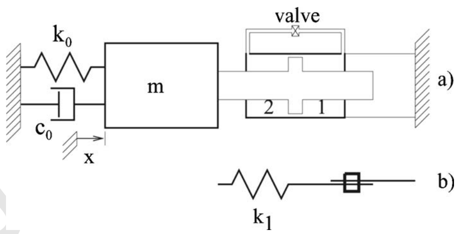

Fig. 1 Schematic representing the variable stiffness device.

perimental results regarding their characteristics, whereas in Sec. 5 we show some simulation results about their feasibility in structural applications.

2 Description of Hardware and Model

The main idea is to find a control law for a device that acts like a spring, whose stiffness can be manipulated in real time, without adding significant amounts of energy. As discussed throughout this paper, we are interested in two forms of control logic: (i) when the stiffness of the device can be switched between zero and the maximum value at appropriate times (i.e., variable stiffness form) or (ii) when the stiffness can be changed from the maximum value to zero and *immediately* increased back to the maximum value (i.e, resetable from).

The basic concept can be demonstrated in the schematic shown in Fig. 1, where we show a simple mass-spring system connected to the proposed device, which is depicted as a double-acting cylinder with an external line that connects the chambers on sides 1 and 2 of the cylinder through an on/off, or a proportional valve. When this valve is closed, motion of the piston compresses the gas, and as shown in [16], the force produced by the gas can be closely approximated by a linear spring with stiffness $k_1 = 2A^2\kappa p_o / v_o$, where A is the piston area, p_o is initial pressure, v_o is the initial volume, and κ is the ratio of constant pressure specific heat of the gas to constant volume specific heat (c_p/c_v) of the gas. It was assumed for this derivation that p_o and v_o are equal on both sides of the piston. The linear spring approximation is represented in the Fig. 1 below the cylinder as spring of stiffness k_1 connected to ground through a collar. When the valve is open, no force is produced by motion of the piston because the gas flows easily between the two sides of the cylinder. This corresponds to the collar being unlocked and sliding freely. Otherwise, closing the valve is analogous to locking the collar with zero force from the spring k_1 at some position $x = x_s$.

Our hardware implementation of this device is shown in Fig. 2. The cylinder is a standard Parker hydraulic cylinder capable of a peak pressure of 5000 psi (34.4 MPa) with a 4 in. (10.16 cm) bore and a 3 in. (7.62 cm) stroke. The valve connecting the two sides of the cylinder is a Moog direct-drive proportional valve capable of <5 ms response times with the orifice area proportional to the control voltage. We filled both sides of the hydraulic cylinder with nitrogen gas up to about $p_o = 800$ lb/in.^{2} (55 atm). Note that standard hydraulic cylinders can handle up to 5000 lb/in.^{2}, so that peak force level of about 30,000 lb can be achieved with this actuator. In Sec. 4, we present preliminary experimental results obtained from several high-capacity devices.

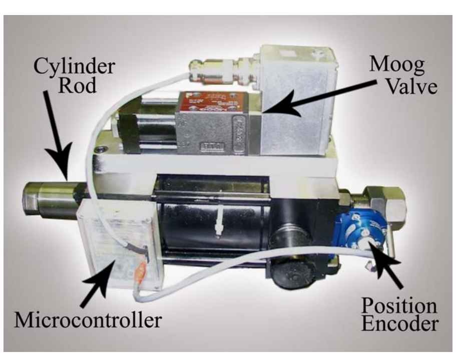

Fig. 2 Variable stiffness device capable of 30,000 lb output

Next, we study ways to control this stiffness value, first for a simple one-degree-of-freedom system, taking the flow dynamics into account, and then in Sec. 3, by using the linear stiffness approximation.

2.1 Optimal Control of a Gas-Filled Actuator. Given the ability to create a variable valve orifice area as the control $u_v(t)$ for this system, we first consider solving the following problem: "Which control extracts energy from the structure most quickly?" Given an initial condition, and/or assuming that a disturbance is known in advance, we can obtain a solution to this problem using tools from optimal control theory. Although it is generally not possible to know what the disturbance or the initial conditions are going to be ahead of time, knowing the optimal solution to this problem sheds light on the form of the feedback control law actually used.

In order to solve the optimal control problem, we first obtain the equations for motion for the structure and gas-filled actuator. For a single degree of freedom system like the one shown in Fig. 1, the equations of motion are

$$m\ddot{x} = -k_o x + (p_2 - p_1)A \tag{1}$$

where k_o is the structural stiffness, A is the area of the piston in the actuator, p_1 and p_2 are the fluid pressures in chambers 1 and 2, and we have neglected viscous damping.

The dynamics of the gas flow and the chamber pressure are found by considering a power balance of the system [20].

$$c_p T\dot{m} - p\dot{v} + \dot{Q} = \frac{c_v}{R}\frac{d}{dt}(pv) \tag{2}$$

where p is the pressure inside the chamber, v is the chamber volume, $\dot{m}$ is the gas mass flow rate into the chamber, T is the gas temperature, R is the universal gas constant, $\dot{Q}$ is the heat transfer rate through the cylinder wall, and c_p, c_v are the gas constant pressure and constant volume specific heats, respectively. In (2), $c_p T\dot{m}$ is the internal energy of the air flowing into the chamber, $p\dot{v}$ is the power output by the moving piston, and $(c_v/R)(d/dt)(pv)$ is the time derivative of the total internal energy of the air in the chamber. We assume $\dot{Q}=0$ because the heat transfer process has a much slower time constant than the air flow dynamics. We rewrite (2) by using $c_p/c_v \equiv \kappa$ and the fact $R = c_p - c_v$, to obtain to a differential equation for gas flow into chambers 1 and 2

$$\dot{p}_1 = \frac{\kappa}{v_1}(RT\dot{m}_1 - p_1\dot{v}_1) \tag{3}$$

$$\dot{p}_2 = \frac{\kappa}{v_2}(RT\dot{m}_2 - p_2\dot{v}_2) \tag{4}$$

The mass flow rates $\dot{m}_1, \dot{m}_2$ are controlled by the proportional valve. As shown experimentally in [20], the flow rates can be approximated reasonably well by

$$\dot{m}_1 = -\dot{m}_2 = c u_v (p_2 - p_1) \tag{5}$$

where c is a constant that depends on the valve orifice area, and u_v is the valve control voltage, which can vary from zero (value closed) to one (valve completely open).

Equations (1)–(5) define the dynamics of the system, and given an initial condition for example, the control $u_v(t)$ that minimizes the mechanical energy can be found. To accomplish this, we solved the following nonlinear optimal control problem:

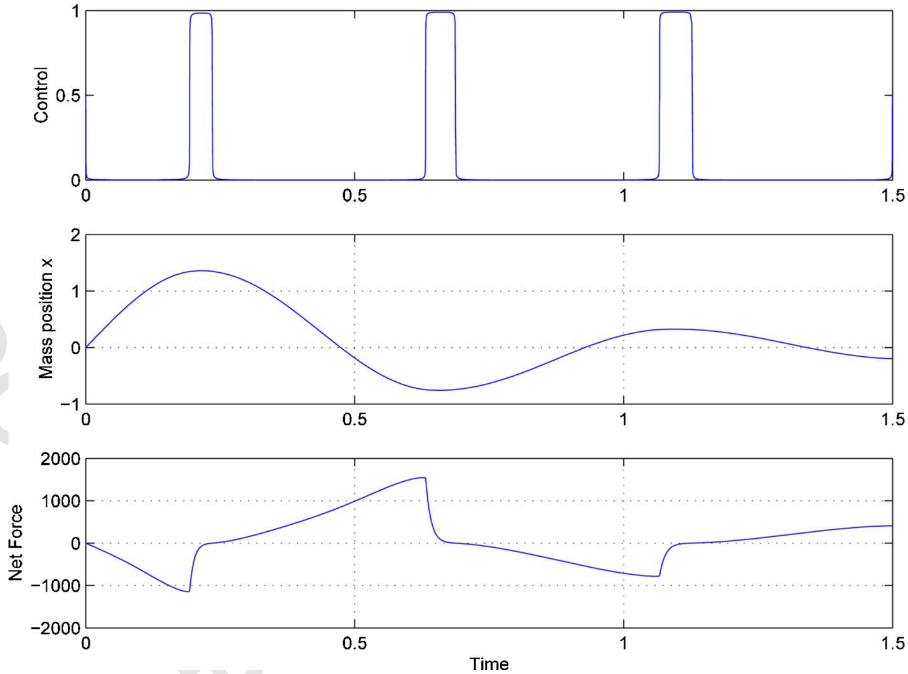

Fig. 3 u_v pulses for a short time while the actuator resets.

$$\underset{u_v(t)}{\text{Min}} J[u_v(t)] = \frac{1}{2}\left\{kx(t_f)^2 + m\dot{x}(t_f)^2 + \int_0^{t_f} \epsilon u_v(t)^2 dt\right\} \quad (6)$$

subject to (1)–(5) and $u_v(t) \in [0,1]$. With t_f fixed and ϵ a small positive constant, we are minimizing the energy in the structure at the final time. The nonzero weighting on the u_v^2 term in (6) was needed for the numerical algorithm used to solve the problem. This term allows the nonlinear problem to be solved via a sequence of manageable linear quadratic subproblems [21]. For ϵ small, the solution has little sensitivity to this parameter. For large t_f, the energy terms outside the integral are easily driven to zero since a small $u_v(t)$ produces a dampinglike effect. As t_f is decreased, at some point the energy terms can no longer be driven to zero for any control $u_v(t) \in [0,1]$. The least time for which the energy terms can be driven to zero is denoted as the minimum time t_f^*. To render the energy minimization reasonable, we are interested in finding solutions for relatively small final times $t_f \le t_f^*$.

Figure 3 shows one sample solution to the control problem in which $t_f \le t_f^*$. Note that the optimal $u_v(t)$ is usually zero, which means that the valve is usually closed so that no gas flows between the two chambers. But at instants when $x(t)$ is maximum or minimum, the optimal $u_v(t)$ pulses to one for a short time. The fact that the control is bang-bang in this manner is also a necessary condition for the optimal control. This is a standard result for minimum time problems for systems that are affine to the control (see, e.g., [22]). Physically, this solution corresponds to keeping the valve closed until the gas in the actuator is most compressed, and opening the valve for a brief time so that the pressure equalizes between the two sides of the cylinder. In doing so, the maximum amount of energy is transformed from the vibrating structure into heat in the cylinder.

Inspection of the time-optimal control and comparison to the idealization as a simple controllable spring element suggests that to maximize the energy transfer, the value is opened (spring is set to zero) at the peak displacement, before some of the stored strain energy is returned to the structure. In Fig. 1, this related to set $k_1=0$ at peak x to remove the energy stored in the spring (when $(1/2)k_1x^2$ is maximum). Similar results (qualitatively) are obtained for different initial conditions or disturbances. As Sec. 3 shows, a passivity approach can be used to obtain similar results, for all possible initial conditions and disturbances, in a feedback form.

3 Design of the Switching Law

As seen in Fig. 3, the optimal approach often results in a switching law that maintains the valve closed most of the time and occasionally opens the valve for short periods of time. In the linear spring analogy, this corresponds to keeping the stiffness at high values most of the time, while occasionally (e.g., at peak displacements) reducing the stiffness to drain energy and restoring or resetting the stiffness to the high value rapidly. In this section, we derive a feedback switching law, based on the linear spring approximation for the actuator, that can be applied to general disturbances, multidegree-of-freedom systems, etc.

At any given time t, we use x_s to denote the position of the piston at the last resetting of the device to its "high" stiffness value; i.e., x_s is a piecewise constant function, whose values are changed due to resetting. For a spring, this corresponds to the setting the unstretched position of the spring to x_s. As a result, the energy stored in the actuator is $(1/2)k_1(x-x_s)^2$; i.e., the energy stored is determined by the compression or extension of the spring is determined from the last resetting time. Adding this to the potential energy of the system (i.e., the structure plus the actuator), and application of Lagrange's equations, leads to the equation of motion

$$m\ddot{x} + [k_o + \alpha(x)k_1]x + c_o\dot{x} = u(t) + \alpha(x)k_1x_s \quad (7)$$

where $\alpha(x)$ is either zero (low stiffness) or one (high stiffness), and thus x_s is the value of $x(t)$ the last time α was set to 1 (or "reset"). Here, u denotes additional inputs due to disturbances.

We note here that the model for *any passive variable stiffness device* must take into account the position x_s for which the change in stiffness occurs. The reasoning for this statement is as follows. Assume the device has two stiffness values, k_{high} and k_{low}. A switch in stiffness from low to high at $x \neq 0$ would require an addition of energy equal to $(1/2)(k_{\text{high}}-k_{\text{low}})x^2$, if x_s is not taken into account. Thus, an injection of energy is needed, and this contradicts the assumption that the device is passive. Some researchers do not include x_s in their models, which will lead to erroneous results. This statement applies to alternative variable stiffness mechanisms such as passive piezoelectric devices.

As with most of the semi-active devices, the resulting system is nonlinear due to the state dependent stiffness. As a result, analytical results in study of stability and performance (e.g, L_2 or energy gain) have been rare. Here, we rely on standard passivity results (see [23]). We use the mechanical energy in the nominal system (i.e., structure without the device) as the storage function

$$V = \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}k_o x^2$$

with the nominal output $y = \dot{x}$, to get

$$\dot{V} = -c_o y^2 + yu + \alpha(x)k_1\dot{x}(x_s - x) \tag{8}$$

where the first two terms on the right-hand side are from the nominal system. Without the last term, following standard steps as in [23], these terms would ensure that the nominal system is strictly output passive (lossless if $c_o=0$); i.e., the rate of energy storage in the system is less than (or for $c_o=0$ equal to) the energy injected by the input u. The last term in (8) is due to the actuator. It is clear that to preserve the passivity of the system (i.e., to avoid increasing the stored energy by this device at any time—i.e., keeping the device semiactive), we need

$$\alpha(x)\dot{x}(x_s - x) \leqslant 0, \qquad \forall\, t \tag{9}$$

Once the passivity is preserved, given that the storage function is positive definite, without external disturbances (i.e., $u=0$) the system is asymptotically stable (for $c_o=0$, a simple application of LaSalles' invariance principle will be needed).

Note that (9) concerns the rate of energy flow into the rest of the system (i.e., the structure) from the actuator, thus a positive sign implies an undesirable direction for energy flow. The fail-safe mechanism here is to ensure that α is set to zero (i.e., stiffness is lowered or valve is opened) if $(x_s - x)\dot{x} \geqslant 0$.

We now introduce the basic switching logic used. From now on, by setting α to zero, we mean setting the stiffness to its lower value (e.g., the valve is opened). A "reset" of the device means α is set to one by increasing the stiffness to its high value (e.g., closing the device). Suppose the device is reset at $t=t_1$. At this time, by definition, $x_s = x(t_1)$. Because of the continuity of $\dot{x}$, there exists a $t_2 > t_1$ such that for $t \in (t_1, t_2]$ the sign of $\dot{x}$ does not change. During this period, we can write $x(t) - x_s = \int_{t_1}^{t}\dot{x}dt$ and thus $\mathrm{sign}[x(t) - x_s] = \mathrm{sign}(\dot{x})$. Therefore, during the time interval after reset that $\dot{x}$ does not change sign, we have $\alpha[x_s - x(t)]\dot{x} \leqslant 0$, (recall α is either zero or one) and the semiactive property [i.e., (9)] holds.

The above discussion implies that the stiffness ideally *should* be reduced (i.e., $\alpha=0$) when $\dot{x}$ changes its sign, though it *can* be reduced at other times as well. Once this is accomplished, the stiffness can be reset to the high value (i.e., $\alpha=1$), i.e., it can be reset, at any time that is physically possible; e.g., as soon as the valve can be closed. Finally, note that as long as $\alpha=1$, and $(x_s - x)\dot{x} < 0$, the actuators are draining energy from the structure and storing it in form of potential energy (to be drained again when $\alpha=0$). Thus, it is desirable to avoid resetting for as long as possible, since energy stored is proportional to the square of the stretched length [i.e., $(1/2)k_1(x_1+x_2)^2 \geqslant (1/2)k_1x_1^2 + 1/2k_1x_2^2$]. As a result, lowering the stiffness with $\alpha=0$ while (9) holds is not desirable, while resetting α to one as soon as possible is desirable.

This leads to the following *ideal resetting* rule, which is a modified form of the switching logic proposed in [16]

$$\alpha = 0 \quad \text{when } \dot{x} \text{ changes sign}$$

$$\alpha = 1 \quad \text{otherwise} \tag{10}$$

The ideal case above assumes that the energy in the actuator can be drained instantaneously. In most practical situations, however, removing energy takes some nonzero duration of time (for example, the plot in Fig. 5 discussed in the next section for the prototypes discussed in this paper). In such cases, we modify (10) to the following:

$$\alpha = 0 \quad \text{when } \dot{x} \text{ changes sign}$$

$$\alpha = 1 \quad \text{as soon as possible} \tag{11}$$

by which we mean that as soon as the energy in the device is drained, set $\alpha=1$. The above development is summarized by the following key technical results:

i. For the system (7), mechanical energy is drained from the system as long as (9) holds.
ii. After any reset, or switch from $\alpha=0$ to $\alpha=1$, there is a time interval for which (9) holds.
iii. Both the ideal (10) and the practical (11) switching rules ensure that (9) holds.

3.1 Control of Multiple Degree-of-Freedom Structures. Next, we generalize this approach to a multidegree-of-freedom systems, in which a number of these devices are installed. For small motion, x_i, the displacement *along* the length of the ith device can be represented by

$$x_i = T_i^T z$$

for some transformation T_i, where z is the vector of generalized coordinates and x_i is the motion along the main axis of the device. The energy stored in the ith device is thus

$$U_i = \frac{1}{2}\alpha_i(z)k_i(x_i - x_{s,i})^2 = \frac{1}{2}(z - z_{s,i})^T T_i[k_i\alpha_i(z)]T_i^T(z - z_{s,i})$$

$$= \frac{1}{2}\alpha_i(z)(z - z_{s,i})^T K_i(z - z_{s,i})$$

where K_i is the contribution of the ith device to the overall stiffness (i.e., $K_i = k_iT_iT_i^T$), with k_i the stiffness of the element and $\alpha_i(\cdot)$ is the switching law. Here, $z_{s,i}$ is the state vector at the last time the ith actuator was reset (i.e., the last time when α_i became 1). Ideally, we seek a decentralized switching law, i.e., $\alpha_i(x_i)$, which is possible as shown below.

After using the above expression for the potential energy in the actuators and applying Lagrange's equations, the equations of motion for the m-degree-of-freedom structure become

$$M\ddot{z} + \left[K_o + \sum \alpha_i(z)K_i\right]z + C_o\dot{z} = \boldsymbol{B}u(t) + \sum \alpha_i(z)K_i z_{s,i}$$

where $\boldsymbol{B}$ is the influence vector associated with disturbances (or other inputs), while M and K_o are the nominal mass and stiffness matrices. Next, we define outputs $y = \boldsymbol{B}^T\dot{z}$, and apply the same approach as before by using the positive definite storage function to be the mechanical energy of the system (without the energy stored in the actuators)

$$V = \tfrac{1}{2}\dot{z}^T M\dot{z} + \tfrac{1}{2}z^T K_o z$$

which yields

$$\dot{V} = -\dot{z}^T C_o\dot{z} + y^T u + \sum \alpha_i(z)\dot{z}^T K_i(z_{s,i} - z)$$

Recalling that $K_i = k_iT_iT_i^T$ and $x_i = T_i z$, we get

$$\dot{V} = -\dot{z}^T C_o\dot{z} + y^T u + \sum \alpha_i(z)k_i\dot{x}_i(x_{s,i} - x_i)$$

Similar to the one-degree-of-freedom system, we seek to design α_i such that the actuators do not increase the rate energy storage in the rest of the system (i.e., the structure). Thus, to preserve the semi-active property, we obtain the same switching logic for each α_i which is the same as (9) for the ith device,

$$\alpha_i(x_i,\dot{x}_i)\dot{x}_i(x_{s,i}-x_i) \leqslant 0, \quad \forall\, t, \quad i=1,2,\ldots,l \qquad (12)$$

which is decentralized and depends only on local coordinates (i.e., motion along the length of the device), and independent of nominal mass and stiffness properties.

If all modes are damped (i.e.,$C_o > 0$), we can write

$$\dot{V} \leqslant -\delta y^T y + y^T u + \sum \alpha_i(x_i)k_i\dot{x}_i(x_{s,i}-x_i)$$

where $\delta = \lambda_{\min}(C_o)/\lambda_{\max}(B^TB)$. Then standard passivity results show that the decentralized switching logic above preserves the estimate for the L_2 or energy gain from u to y (i.e., $1/\delta$) and asymptotic stability of the system (in the absence of external disturbances), under mild controllability or observability conditions.

As discussed in [16], when damping matrix is not positive definite, asymptotic stability is not necessarily guaranteed and the state vector converges to the intersection of sets or manifolds $\dot{z}^T C_o \dot{z}=0$ and $\dot{z}^T K_i \dot{z}=0$. In such cases, zero-state observability with K_i or similar concepts may be used to establish asymptotic stability, though depending on C_o and location of the devices (i.e., structure of K_i) the system may be stable only.

Remark. The switching law above was developed by defining $y=B^T\dot{z}$, to exploit the passivity framework and to establish the semiactive nature of the switching law. In practice, other (additional or different) outputs may be used for different purposes, without compromising the semi-active property as long as the variables needed for the switching law are measured. For example, implementing the switching law in (12) requires, at a minimum, $\dot{x}_i$. Also note that we have assumed continuity of the differential equations for the structural motion. This is a relatively mild assumption and is met in all realistic cases (to ensure chattering is avoided, one can introduce a small threshold in the control logic).

3.2 Variable Stiffness Feedback Control. Let us now review and compare the resetting approached discussed above with a simple variable stiffness technique where it is assumed that the actuator can operate at two distinct stiffness values. In general, this leads to a system for which a variety of results from variable structure or switched systems can used.

In this case, the equations of motion for the one-degree-of-freedom system is

$$m\ddot{x} + [k_o + \alpha(x)k_1]x + c_o\dot{x} = u(t) \qquad (13)$$

where c_o, $u(t)$, k_1 are as in (7), and $\alpha(x)$ is the switching law that controls the stiffness of the device. Note that here the device alters the stiffness only (ie., no x_s). We also assume that it is possible to develop devices that allow all stiffness values between zero and k_1 (i.e., $1 \geqslant \alpha(x) \geqslant 0$).

At a given deformation, increasing the stiffness of a spring requires the input of energy unless it is done at its unstretched position. Since we are interested in developing a low-power or semiactive device, this issue plays an important role in developing control logic for this device. As before, we start with a storage function

$$V = \tfrac{1}{2}m\dot{x}^2 + \tfrac{1}{2}k_o x^2$$

i.e., the mechanical energy in the nominal system. It is easy to see that with velocity measurement, i.e., $y=\dot{x}$, we have

$$\dot{V} = -c_o y^2 + yu - \alpha(x)k_1\dot{x}x$$

The first two terms on the right-hand side are from the nominal system and establish passivity and stability of the nominal system (similar to the resetting case). It is clear that in order not to alter the passivity of the system (e.g., to avoid increasing the stored energy at any time), thus satisfying the basic property of the semi-active approach, we need $\alpha(x)k_1\dot{x}x \geqslant 0$. Given the range of values for $\alpha(x)$, the resulting semi-active switching law becomes

$$\alpha(x,\dot{x}) = 1 \quad \text{if } \dot{x}x \geqslant 0$$

$$\alpha(x,\dot{x}) = 0 \quad \text{if } \dot{x}x < 0 \qquad (14)$$

that is, given the desire to remove as much energy as possible yields an "on-off" or two-state logic even if intermediate values of α were feasible. Also, passivity properties of the nominal system is preserved and following standard steps, we can show that stability and L_2 gain of the nominal system is preserved, as well. The generalization to multidegree-of-freedom system follows exactly as before, leading to a decentralized control law of (14). For brevity, the details are omitted.

3.3 Comparison and Discussion. The switching law (14) and approaches similar to it, have been used before. For example, [9] used a similar logic for a single degree of freedom system to demonstrate a simple variable structure system, whereas [1,2,10] had used variable stiffness devices to move energy to different modes, depending the excitation. In particular, [10] included a discussion on changing the stiffness to high values at zero deflections. More recently, the variable stiffness approach has been used by by Patten and co-workers (e.g., [3]) and Dawson and co-workers (e.g., [13], when the stiffness is altered with piezoactuators). Typically, the stiffness is increased to the higher value according to a logic similar to (14).

The resetting method coincides with the variable stiffness approach if we wait and reset (i.e, setting the stiffness to high) only when $x(t)=0$, which results in $x_s=0$. In such a case, the device is not in operation, and thus is not collecting energy, during the period of time from reset and when $x(t)$ crosses zero. This implies that the resetting approach is often more effective than variable stiffness since it is collecting energy, to be drained at peak storage, at all times, whereas the variable stiffness device is "off" roughly half the time. For results regarding rate decay (in simple first-order systems), or placement of devices (in MDOF structures), one can consult Ref. [11,16], respectively.

Remark. In [24], the term "reset control" is used to address a generalization of the Clegg integral from the 1950s, which has shown benefits in improving overshoot properties of linear controllers. There are similarities between these approaches, in the sense the equations of motion here can be presented as a special case of the model used there, and the devices discussed here have shown strong overshoot suppression properties (see [12]). The reset control of [24], however, is a modification to a traditional (active) compensator, whereas the reset logic discussed here is vibration suppression device that is added to the structure or can be combined with a variety of other actuators, if desired (in which case the switched or hybrid systems approach might be an appropriate framework). Also, the passivity approach has led to stability and performance guarantees in relatively simple steps, consistent with the suggested future work in [24].

4 Preliminary Experimental Results

Figure 4 shows the behavior of a prototype, obtained from shaking table testing at the National Center for Research on Earthquake Engineering in Taiwan (see [19] for a more comprehensive description and additional results). Here, the device is subjected to sinusoidal motion with peak to peak distance of 20 mm, with a peak resisting force of 30 kN. The sudden drop in Fig. 4(*b*) corresponds to resetting of the actuator, when the valve is opened at the extreme end of the motion to drain energy and reset the effective stiffness to zero. This is more pronounced in the hysteresis plot in Fig. 4(*c*), where at each extreme end of the motion, resetting reduces the stiffness and thus the energy stored in the device. Also, note that Fig. 4(*c*) shows that the effective stiffness is quite close to a linear spring (as used in the development of Sec. 3) throughout the range of motion.

Given the scale used in Fig. 4, it is difficult to estimate the amount of time it takes to drain the energy and reset the actuator. Figure 5 gives a more detailed look at the response of the resetting controller, operating in an experimental single-degree-of-freedom test apparatus, subjected to initial displacement, in a setup quite

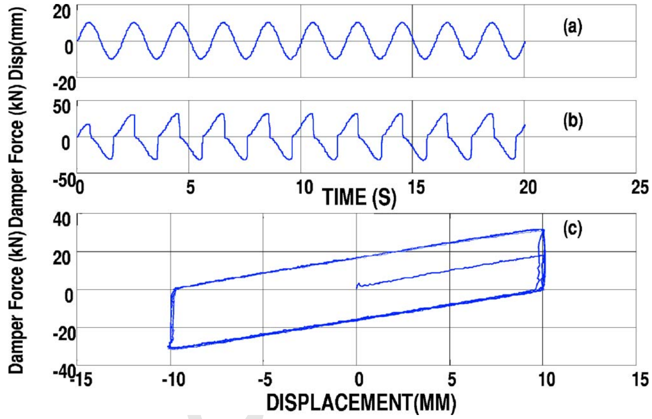

Fig. 4 (***a***) **Piston displacement versus time,** (***b***) **net actuator force versus time, and** (***c***) **force versus diplacement**

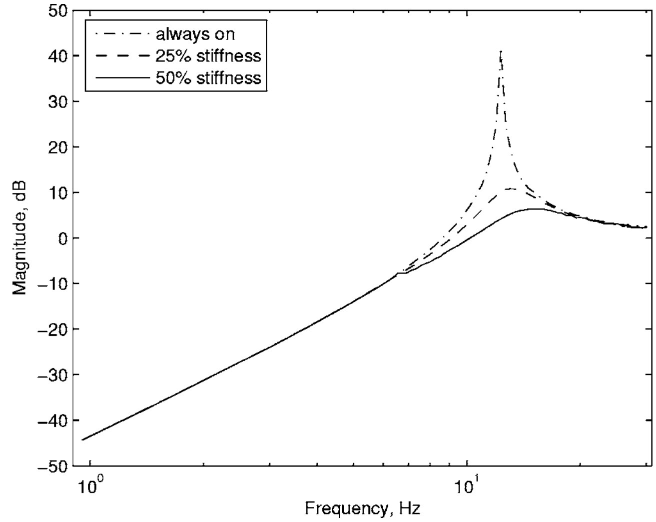

Fig. 6 Response of a single actuator to sine-wave base motion

similar to the schematic in Fig. 1. The plot shows the time it takes for the actuator force to reach equilibrium (i.e., all stored energy is drained) after the valve is opened fully. The signal labeled *net force on piston* is the net force exerted by the gas on the piston. Within the three narrow bands seen in Fig. 5, the valve is commanded to fully open, while outside these bands the valve is commanded to fully close. Note that within these bands, the force exerted by the gas on the piston decays to zero, taking 30–40 ms. Also note that the controller detects peaks in the position of the piston, and initiates the resetting (i.e., closing the valve). As discussed earlier, from an energy standpoint, we would like to open the valve when the force reaches a peak. The shape of the plot reflects the fast decay in motion in the free response case. Further experimental results are presented in [18,19].

The results here support the main characteristics used in developing the results of Sec. 3: the validity of using a linear spring to approximate the behavior of a closed cylinder over a wide range of peak forces (at least up to 30,000 kN), the existence of modest delays in closing the valve, consistent with the control logic discussed earlier, and the feasibility of the concept for large-scale devices.

5 Performance Comparison and Benchmark Simulations

To evaluate the effects of resetting devices, the following comparison is made. In a one-degree-of-freedom system, similar to the schematic of Fig. 1, we introduce a base motion in the form of a simple sine-wave and obtain the magnitude of the resulting motion (similar to moving one of the side walls in Fig. 1 and measuring the displacement of the mass). By sweeping through frequencies, we can obtain a form of frequency response for the device. Note that in general, semi-active devices are nonlinear (in this case, stiffness is state dependent) and notions of frequency response should be used carefully. The resetting technique, as discussed in [25], has the homogeneity property that results in magnitude-independent frequency response, unlike many other semi-active techniques (assuming no physical limits on the stroke of the device).

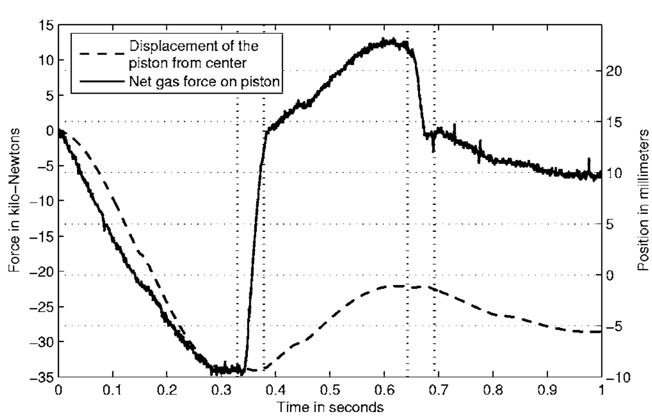

Fig. 5 Resetting response of a single actuator

The results are presented in Fig. 6, where the dashed-dotted line ("always on") is the frequency response of the system if the stiffness of device is simply added to the overall stiffness by preventing any resetting. The other plots correspond to the cases where the stiffness of the resetting element (which is turned on) is 25% or 50% of the total stiffness available. The 25% level might be more practical, and provides significant attenuation. The 50% case shows a drastic reduction in response consistent with the results forming suppressing vibration due to initial conditions, discussed in [11]. Overall, it shows the main benefit to be precisely in the critical frequency ranges.

Among the advantages of a variable stiffness device for extracting energy from the structure, as opposed to damping devices, is that in cases of shock loading, large forces are not transmitted to the structure. This is because high velocities create large forces in traditional dampers, but create no force in the variable stiffness device (see [12] for an example application to an automotive suspension where the force transmitted through a conventional damper is more than an order of magnitude higher than the force transmitted through the resetting device). Here, we compare the performance of the resetting approach to that of an MR damper using the model developed in [8], where the NS component of the 1940 El Centro earthquake was the input to a three-story structure. For the same structure, we simulate the results of placing a single resetable device between the first and second floors. The device has an effective stiffness of about 9 kN/cm. In Table 1, we show the peak displacement (x_i) of each story relative to ground, the peak interstory drifts (d_i), the peak absolute acceleration of each story (a_{ia}), and the peak force (f) for the uncontrolled systems as well as those obtained with either an MR damper or a resetable device. The controller used for the MR device is the so-called clipped optimal control (i.e., an optimal control law, such as LQR or H_2, which is clipped if the device cannot provide the maximum forces needed by the controller). As discussed in [8], this rather complex and centralized approach often results in the best performance in ER- and MR-based approaches.

As Table 1 shows, the performance of the two devices are quite

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Table 1 Effects on a three-story building

	Uncontr.	Clipped opt. (MR)	Resetting
x_1 (cm)	0.20	0.04	0.04
x_2 (cm)	0.31	0.07	0.08
x_3 (cm)	0.36	0.10	0.12
a_1 (cm/s^2)	421	341	363
a_1 (cm/s^2)	430	363	318
a_1 (cm/s^2)	571	341	340
f (N)	0	492	470
d_1 (cm)	0.20	0.04	0.04
d_2 (cm)	0.11	0.04	0.03
d_3 (cm)	0.05	0.03	0.03

similar, and both deliver significant improvements from the open-loop or uncontrolled case. This is not unexpected, since several studies (see [8] and references therein) have shown similar patterns; a relatively large number of devices with roughly equal capacity (e.g., maximum resistive force) showing more or less similar results. Generally, the resetable devices perform better for higher-frequency disturbances (recall the discussions on their benefits in shock-type disturbances). Overall, these devices offer similar performance at far lower complexity (e.g., decentralized logic) with standard and reliable hydraulic technologies. More extensive comparisons can be found in [17], in which a variety of semi-active devices are compared on this benchmark.

6 Conclusions

We have shown, using an optimal control approach, that the resetting techniques is the fastest method for removing energy from a vibrating structure, using variable stiffness actuators. We developed a feedback control law, based on passivity arguments, that implements the optimal control and extends previous results to account for switching delays in practical hardware. Finally, we have presented experimental and simulation results that demonstrate that resetting is a viable method for applications in full scale structures.

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